19

Determinants

TOPIC 1 **Minor & Co-factor of an Element of a Determinant, Value of a Determinant, Property of Determinant of Matrices, Singular & Non-Singular Matrices, Multiplication of two Determinants**

1. Let $\theta = \frac{\pi}{5}$ and $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$. If $B = A + A^4$, then det (B): **[Sep. 06, 2020 (II)]**

(a) is one (b) lies in (2, 3)
(c) is zero (d) lies in (1, 2)

2. If $\Delta = \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 2x-3 & 3x-4 & 4x-5 \\ 3x-5 & 5x-8 & 10x-17 \end{vmatrix} = Ax^3 + Bx^2 + Cx + D,$

then $B + C$ is equal to : **[Sep. 03, 2020 (I)]**

(a) –1 (b) 1 (c) –3 (d) 9

3. Let $a - 2b + c = 1$.

If $f(x) = \begin{vmatrix} x+a & x+2 & x+1 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$, then : **[Jan. 9, 2020 (II)]**

(a) $f(-50) = 501$ (b) $f(-50) = -1$
(c) $f(50) = -501$ (d) $f(50) = 1$

4. If $\Delta_1 = \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ and

$\Delta_2 = \begin{vmatrix} x & \sin 2\theta & \cos 2\theta \\ -\sin 2\theta & -x & 1 \\ \cos 2\theta & 1 & x \end{vmatrix}, x \neq 0$;

then for all $\theta \in \left(0, \frac{\pi}{2}\right)$: **[April 10, 2019 (I)]**

(a) $\Delta_1 - \Delta_2 = -2x^3$
(b) $\Delta_1 - \Delta_2 = x(\cos 2\theta - \cos 4\theta)$
(c) $\Delta_1 \times \Delta_2 = -2(x^3 + x - 1)$
(d) $\Delta_1 + \Delta_2 = -2x^3$

5. The sum of the real roots of the equation

$\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0,$ is equal to : **[April 10, 2019 (II)]**

(a) 6 (b) 0 (c) 1 (d) –4

6. Let $A = \begin{bmatrix} 2 & b & 1 \\ b & b^2+1 & b \\ 1 & b & 2 \end{bmatrix}$ where $b > 0$. Then the minimum value of $\frac{\det(A)}{b}$ is: **[Jan. 10, 2019 (II)]**

(a) $2\sqrt{3}$ (n) $-2\sqrt{3}$ (c) $-\sqrt{3}$ (d) $\sqrt{3}$

7. If $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A+Bx)(x-A)^2$, then the ordered pair (A, B) is equal to : **[2018]**

(a) (–4, 3) (b) (–4, 5)
(c) (4, 5) (d) (–4, –5)

8. If $S = \left\{ x \in [0, 2\pi] : \begin{vmatrix} 0 & \cos x & -\sin x \\ \sin x & 0 & \cos x \\ \cos x & \sin x & 0 \end{vmatrix} = 0 \right\}$, then

$\sum_{x \in S} \tan\left(\frac{\pi}{3} + x\right)$ is equal to **[Online April 8, 2017]**

(a) $4 + 2\sqrt{3}$ (b) $-2 + \sqrt{3}$
(c) $-2 - \sqrt{3}$ (d) $-4 - 2\sqrt{3}$

9. If $A = \begin{bmatrix} -4 & -1 \\ 3 & 1 \end{bmatrix}$, then the determinant of the matrix $(A^{2016} - 2A^{2015} - A^{2014})$ is : **[Online April 10, 2016]**

(a) –175 (b) 2014 (c) 2016 (d) –25

10. if $\begin{vmatrix} x^2+x & x+1 & x-2 \\ 2x^2+3x-1 & 3x & 3x-3 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix} = ax - 12$, then '$a$' is equal to : **[Online April 11, 2015]**

(a) 24 (b) –12 (c) –24 (d) 12

11. The least value of the product xyz for which the determinant $\begin{vmatrix} x & 1 & 1 \\ 1 & y & 1 \\ 1 & 1 & z \end{vmatrix}$ is non-negative, is : **[Online April 10, 2015]**

(a) $-2\sqrt{2}$ (b) -1

(c) $-16\sqrt{2}$ (d) -8

12. If $f(\theta) = \begin{vmatrix} 1 & \cos\theta & 1 \\ -\sin\theta & 1 & -\cos\theta \\ -1 & \sin\theta & 1 \end{vmatrix}$ and A and B are respectively the maximum and the minimum values of f(θ), then (A, B) is equal to: **[Online April 12, 2014]**

(a) $(3, -1)$ (b) $\left(4, 2-\sqrt{2}\right)$

(c) $\left(2+\sqrt{2}, 2-\sqrt{2}\right)$ (d) $\left(2+\sqrt{2}, -1\right)$

13. If B is a 3 × 3 matrix such that $B^2 = 0$, then det. $[(I + B)^{50} - 50B]$ is equal to: **[Online April 9, 2014]**

(a) 1 (b) 2 (c) 3 (d) 50

14. Let $S = \left\{ \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} : a_{ij} \in \{0,1,2\}, a_{11} = a_{22} \right\}$

Then the number of non-singular matrices in the set S is : **[Online April 25, 2013]**

(a) 27 (b) 24

(c) 10 (d) 20

15. Let A, other than I or – I, be a 2 × 2 real matrix such that $A^2 = I$, I being the unit matrix. Let Tr (A) be the sum of diagonal elements of A. **[Online April 23, 2013]**

Statement-1: Tr (A) = 0

Statement-2: det (A) = – 1

(a) Statement-1 is true; Statement-2 is false.

(b) Statement-1 is true; Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1.

(c) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(d) Statement-1 is false; Statement-2 is true.

16. **Statement - 1:** **[2011RS]**

Determinant of a skew-symmetric matrix of order 3 is zero.

Statement - 2 :

For any matrix A, det $(A)^T$ = det (A) and det (– A) = – det (A).

Where det (B) denotes the determinant of matrix B. Then :

(a) Both statements are true

(b) Both statements are false

(c) Statement-1 is false and statement-2 is true

(d) Statement-1 is true and statement-2 is false

17. Let A be a 2 × 2 matrix with non-zero entries and let $A^2 = I$, where I is 2 × 2 identity matrix. Define

Tr(A) = sum of diagonal elements of A and

$|A|$ = determinant of matrix A.

Statement - 1 : Tr(A) = 0.

Statement -2 : $|A| = 1$. **[2010]**

(a) Statement-1 is true, Statement-2 is true ; Statement-2 is **not** a correct explanation for Statement -1.

(b) Statement -1 is true, Statement -2 is false.

(c) Statement -1 is false, Statement -2 is true .

(d) Statement - 1 is true, Statement 2 is true ; Statement -2 is a correct explanation for Statement -1.

18. Let A be a 2 × 2 matrix with real entries. Let I be the 2 × 2 identity matrix. Denote by tr(A), the sum of diagonal entries of a. Assume that $A^2 = I$. **[2008]**

Statement-1 : If $A \neq I$ and $A \neq -I$, then det $(A) = -1$

Statement-2 : If $A \neq I$ and $A \neq -I$, then tr $(A) \neq 0$.

(a) Statement -1 is false, Statement-2 is true

(b) Statement -1 is true, Statement-2 is true; Statement -2 is a correct explanation for Statement-1

(c) Statement -1 is true, Statement-2 is true; Statement -2 is not a correct explanation for Statement-1

(d) Statement -1 is true, Statement-2 is false

19. Let $A = \begin{vmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{vmatrix}$. If $\left|A^2\right| = 25$, then $|\alpha|$ equals **[2007]**

(a) 1/5 (b) 5

(c) 5^2 (d) 1

20. If $1, \omega, \omega^2$ are the cube roots of unity, then

$\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix}$ is equal to **[2003]**

(a) ω^2 (b) 0

(c) 1 (d) ω

TOPIC 2 Properties of Determinants, Area of a Triangle

21. If the minimum and the maximum values of the function $f:\left[\frac{\pi}{4},\frac{\pi}{2}\right]\to R$, defined by

$$f(\theta)=\begin{vmatrix}-\sin^2\theta & -1-\sin^2\theta & 1\\ -\cos^2\theta & -1-\cos^2\theta & 1\\ 12 & 10 & -2\end{vmatrix}$$ are m and M respectively, then the ordered pair (m, M) is equal to : **[Sep. 05, 2020 (I)]**

(a) $(0, 2\sqrt{2})$ (b) $(-4, 0)$
(c) $(-4, 4)$ (d) $(0, 4)$

22. If $a+x=b+y=c+z+1$, where a, b, c, x, y, z are non-zero distinct real numbers, then $\begin{vmatrix}x & a+y & x+a\\ y & b+y & y+b\\ z & c+y & z+c\end{vmatrix}$ is equal to : **[Sep. 05, 2020 (II)]**

(a) $y(b-a)$ (b) $y(a-b)$
(c) 0 (d) $y(a-c)$

23. Let two points be $A(1,-1)$ and $B(0,2)$. If a point $P(x', y')$ be such that the area of $\Delta PAB = 5$ sq. units and it lies on the line, $3x+y-4\lambda=0$, then a value of λ is: **[Jan. 8, 2020 (I)]**

(a) 4 (b) 3 (c) 1 (d) –3

24. Let $A=[a_{ij}]$ and $B=[b_{ij}]$ be two 3×3 real matrices such that $b_{ij}=(3)^{(i+j-2)}a_{ij}$, where $i, j = 1, 2, 3$. If the determinant of B is 81, then the determinant of A is: **[Jan. 7, 2020 (II)]**

(a) 1/3 (b) 3 (c) 1/81 (d) 1/9

25. A value of $\theta\in(0, \pi/3)$, for which

$$\begin{vmatrix}1+\cos^2\theta & \sin^2\theta & 4\cos 6\theta\\ \cos^2\theta & 1+\sin^2\theta & 4\cos 6\theta\\ \cos^2\theta & \sin^2\theta & 1+4\cos 6\theta\end{vmatrix}=0, \text{ is :}$$

[April 12, 2019 (II)]

(a) $\frac{\pi}{9}$ (b) $\frac{\pi}{18}$ (c) $\frac{7\pi}{24}$ (d) $\frac{7\pi}{36}$

26. Let α and β be the roots of the equation $x^2+x+1=0$. Then for y ‴ 0 in R, $\begin{vmatrix}y+1 & \alpha & \beta\\ \alpha & y+\beta & 1\\ \beta & 1 & y+\alpha\end{vmatrix}$ is equal to: **[April 09, 2019 (I)]**

(a) $y(y^2-1)$ (b) $y(y^2-3)$
(c) y^3 (d) y^3-1

27. Let the numbers 2, b, c be in an A.P. and $A=\begin{bmatrix}1 & 1 & 1\\ 2 & b & c\\ 4 & b^2 & c^2\end{bmatrix}$. If det(A)?[2, 16], then c lies in the interval : **[April 08, 2019 (II)]**

(a) $[2, 3)$ (b) $(2+2^{3/4}, 4)$
(c) $[4, 6]$ (d) $[3, 2+2^{3/4}]$

28. If $A=\begin{bmatrix}1 & \sin\theta & 1\\ -\sin\theta & 1 & \sin\theta\\ -1 & -\sin\theta & 1\end{bmatrix}$; then for all $\theta\in\left(\frac{3\pi}{4},\frac{5\pi}{4}\right)$, det (A) lies in the interval : **[Jan. 12, 2019 (II)]**

(a) $\left(1,\frac{5}{2}\right]$ (b) $\left[\frac{5}{2},4\right)$ (c) $\left(0,\frac{3}{2}\right]$ (d) $\left(\frac{3}{2},3\right]$

29. If $\begin{vmatrix}a-b-c & 2a & 2a\\ 2b & b-c-a & 2b\\ 2c & 2c & c-a-b\end{vmatrix}$ $=(a+b+c)(x+a+b+c)^2$, $x\neq 0$ and $a+b+c\neq 0$, then x is equal to : **[Jan. 11, 2019 (II)]**

(a) abc (b) $-(a+b+c)$
(c) $2(a+b+c)$ (d) $-2(a+b+c)$

30. Let $d\in\mathbf{R}$, and

$$A=\begin{bmatrix}-2 & 4+d & (\sin\theta)^{-2}\\ 1 & (\sin\theta)+2 & d\\ 5 & (2\sin\theta)-d & (-\sin\theta)+2+2d\end{bmatrix},$$

$\theta\in[0, 2\pi]$. If the minimum value of det (A) is 8, then a value of d is: **[Jan 10, 2019 (I)]**

(a) -5 (b) -7
(c) $2(\sqrt{2}+1)$ (d) $2(\sqrt{2}+2)$

31. Let $a_1, a_2, a_3, ..., a_{10}$ be in G.P. with $a_i>0$ for $i = 1, 2, ..., 10$ and S be the set of pairs (r, k), r, k∈N (the set of natural numbers) for which

$$\begin{vmatrix}\log_e a_1^r a_2^k & \log_e a_2^r a_3^k & \log_e a_3^r a_4^k\\ \log_e a_4^r a_5^k & \log_e a_5^r a_6^k & \log_e a_6^r a_7^k\\ \log_e a_7^r a_8^k & \log_e a_8^r a_9^k & \log_e a_9^r a_{10}^k\end{vmatrix}=0$$

Then the number of elements in S, is : **[Jan. 10, 2019 (II)]**

(a) 4 (b) infinitely many
(c) 2 (d) 10

32. If

$$A=\begin{bmatrix} e^t & e^{-t}\cos t & e^{-t}\sin t \\ e^t & -e^{-t}\cos t-e^{-t}\sin t & -e^{-t}\sin t+e^{-t}\cos t \\ e^t & 2e^{-t}\sin t & -2e^{-t}\cos t \end{bmatrix}$$

then A is: **[Jan. 09, 2019 (II)]**

(a) invertible for all $t\in\mathbf{R}$.
(b) invertible only if $t=\pi$.
(c) not invertible for any $t\in\mathbf{R}$.
(d) invertible only if $t=\frac{\pi}{2}$.

33. Let k be an integer such that triangle with vertices (k, –3k), (5, k) and (–k, 2) has area 28 sq. units. Then the orthocentre of this triangle is at the point : **[2017]**

(a) $\left(2,\frac{1}{2}\right)$ (b) $\left(2,-\frac{1}{2}\right)$ (c) $\left(1,\frac{3}{4}\right)$ (d) $\left(1,-\frac{3}{4}\right)$

34. Let ω be a complex number such that $2\omega+1=z$ where $z=\sqrt{-3}$. If $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2-1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix}=3k$, then k is equal to : **[2017]**

(a) 1 (b) –z (c) z (d) –1

35. The number of distinct real roots of the equaiton, $\begin{vmatrix} \cos x & \sin x & \sin x \\ \sin x & \cos x & \sin x \\ \sin x & \sin x & \cos x \end{vmatrix}=0$ in the interval $\left[-\frac{\pi}{4},\frac{\pi}{4}\right]$ is : **[Online April 9, 2016]**

(a) 1 (b) 4 (c) 2 (d) 3

36. If $\alpha,\beta\neq 0$, and $f(n)=\alpha^n+\beta^n$ and

$$\begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix}=K(1-\alpha)^2(1-\beta)^2(\alpha-\beta)^2,$$

then K is equal to: **[2014]**

(a) 1 (b) –1 (c) αβ (d) $\frac{1}{\alpha\beta}$

37. If $\Delta_r=\begin{vmatrix} r & 2r-1 & 3r-2 \\ \frac{n}{2} & n-1 & a \\ \frac{1}{2}n(n-1) & (n-1)^2 & \frac{1}{2}(n-1)(3n-4) \end{vmatrix}$

then the value of $\sum_{r=1}^{n-1}\Delta_r$ **[Online April 19, 2014]**

(a) depends only on a
(b) depends only on n
(c) depends both on a and n
(d) is independent of both a and n

38. If

$$\begin{vmatrix} a^2 & b^2 & c^2 \\ (a+\lambda)^2 & (b+\lambda)^2 & (c+\lambda)^2 \\ (a-\lambda)^2 & (b-\lambda)^2 & (c-\lambda)^2 \end{vmatrix}=k\lambda\begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix},\lambda\neq 0$$

then k is equal to: **[Online April 12, 2014]**

(a) $4\lambda abc$ (b) $-4\lambda abc$ (c) $4\lambda^2$ (d) $-4\lambda^2$

39. If *a*, *b*, *c* are sides of a scalene triangle, then the value of $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is : **[Online April 9, 2013]**

(a) non - negative (b) negative
(c) positive (d) non-positive

40. If *a*, *b*, *c*, are non zero complex numbers satisfying $a^2+b^2+c^2=0$ and

$$\begin{vmatrix} b^2+c^2 & ab & ac \\ ab & c^2+a^2 & bc \\ ac & bc & a^2+b^2 \end{vmatrix}=ka^2b^2c^2,$$ then k is equal to

[Online May 19, 2012]

(a) 1 (b) 3 (c) 4 (d) 2

41. If $\begin{vmatrix} -2a & a+b & a+c \\ b+a & -2b & b+c \\ c+a & b+c & -2c \end{vmatrix}=\alpha(a+b)(b+c)(c+a)\neq 0$

then α is equal to **[Online May 12, 2012]**

(a) $a+b+c$ (b) abc
(c) 4 (d) 1

42. The area of the triangle whose vertices are complex numbers $z, iz, z+iz$ in the Argand diagram is **[Online May 12, 2012]**

(a) $2|z|^2$ (b) $1/2|z|^2$ (c) $4|z|^2$ (d) $|z|^2$

43. The area of triangle formed by the lines joining the vertex of the parabola, $x^2=8y$, to the extremities of its latus rectum is **[Online May 12, 2012]**

(a) 2 (b) 8 (c) 1 (d) 4

44. Let *a*, *b*, *c* be such that $b(a+c)\neq 0$ if **[2009]**

$$\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix}+\begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^n c \end{vmatrix}=0,$$

then the value of *n* is :

(a) any even integer (b) any odd integer
(c) any integer (d) zero

45. If $D=\begin{vmatrix}1 & 1 & 1\\ 1 & 1+x & 1\\ 1 & 1 & 1+y\end{vmatrix}$ for $x \neq 0, y \neq 0$, then D is

(a) divisible by x but not y **[2007]**
(b) divisible by y but not x
(c) divisible by neither x nor y
(d) divisible by both x and y

46. If a_1, a_2, a_3, ..., a_n, ... are in G. P., then the determinant

$$\Delta = \begin{vmatrix}\log a_n & \log a_{n+1} & \log a_{n+2}\\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5}\\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8}\end{vmatrix}$$

is equal to **[2005]**

(a) 1 (b) 0 (c) 4 (d) 2

47. If $a^2+b^2+c^2=-2$ and **[2005]**

$$f(x)=\begin{vmatrix}1+a^2x & (1+b^2)x & (1+c^2)x\\ (1+a^2)x & 1+b^2x & (1+c^2)x\\ (1+a^2)x & (1+b^2)x & 1+c^2x\end{vmatrix},$$

then f(x) is a polynomial of degree

(a) 1 (b) 0 (c) 3 (d) 2

48. If $a_1, a_2, a_3, ..., a_n, ...$ are in G.P., then the value of the determinant **[2004]**

$$\begin{vmatrix}\log a_n & \log a_{n+1} & \log a_{n+2}\\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5}\\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8}\end{vmatrix}, \text{ is}$$

(a) –2 (b) 1
(c) 2 (d) 0

49. If $a>0$ and discriminant of $ax^2+2bx+c$ is –ve, then

$\begin{vmatrix}a & b & ax+b\\ b & c & bx+c\\ ax+b & bx+c & 0\end{vmatrix}$ *is* equal to **[2002]**

(a) +ve (b) $(ac-b^2)(ax^2+2bx+c)$
(c) –ve (d) 0

50. l, m, n are the p^{th}, q^{th} and r^{th} term of a G. P. all positive,

then $\begin{vmatrix}\log l & p & 1\\ \log m & q & 1\\ \log n & r & 1\end{vmatrix}$ equals **[2002]**

(a) –1 (b) 2
(c) 1 (d) 0

TOPIC 3 Adjoint of a Matrix, Inverse of a Matrix, Some Special Cases of Matrix, Rank of a Matrix

51. Let A be a 3×3 matrix such that adj $A=\begin{bmatrix}2 & -1 & 1\\ -1 & 0 & 2\\ 1 & -2 & -1\end{bmatrix}$ and $B=\text{adj}(\text{adj} A)$. If $|A|=\lambda$ and $|(B^{-1})^T|=\mu$, then the ordered pair, $(|\lambda|, \mu)$ is equal to : **[Sep. 03, 2020 (II)]**

(a) $\left(3, \frac{1}{81}\right)$ (b) $\left(9, \frac{1}{9}\right)$

(c) $(3, 81)$ (d) $\left(9, \frac{1}{81}\right)$

52. If the matrices $A=\begin{bmatrix}1 & 1 & 2\\ 1 & 3 & 4\\ 1 & -1 & 3\end{bmatrix}$, $B=\text{adj } A$ and $C=3A$, then $\frac{|\text{adj } B|}{|C|}$ is equal to : **[Jan. 9, 2020 (I)]**

(a) 8 (b) 16 (c) 72 (d) 2

53. If $B=\begin{bmatrix}5 & 2\alpha & 1\\ 0 & 2 & 1\\ \alpha & 3 & -1\end{bmatrix}$ is the inverse of a 3×3 matrix A, then the sum of all values of α for which det $(A)+1=0$, is : **[April 12, 2019 (I)]**

(a) 0 (b) –1 (c) 1 (d) 2

54. If $\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix} \cdot \begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix} \cdot \begin{bmatrix}1 & 3\\ 0 & 1\end{bmatrix} \cdots\cdots \begin{bmatrix}1 & n-1\\ 0 & 1\end{bmatrix} = \begin{bmatrix}1 & 78\\ 0 & 1\end{bmatrix}$, then the inverse of $\begin{bmatrix}1 & n\\ 0 & 1\end{bmatrix}$ is: **[April 09, 2019 (II)]**

(a) $\begin{bmatrix}1 & 0\\ 12 & 1\end{bmatrix}$ (b) $\begin{bmatrix}1 & -13\\ 0 & 1\end{bmatrix}$

(c) $\begin{bmatrix}1 & -12\\ 0 & 1\end{bmatrix}$ (d) $\begin{bmatrix}1 & 0\\ 13 & 1\end{bmatrix}$

55. Let A and B be two invertible matrices of order 3×3. If det $(ABA^T)=8$ and det $(AB^{-1})=8$, then det $(BA^{-1}B^T)$ is equal to : **[Jan. 11, 2019 (II)]**

(a) $\frac{1}{4}$ (b) 1

(c) $\frac{1}{16}$ (d) 16

56. If $A=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$, then the matrix A^{-50} when $\theta=\frac{\pi}{12}$, is equal to: **[Jan 09, 2019 (I)]**

(a) $\begin{bmatrix}\frac{1}{2} & -\frac{\sqrt{3}}{2}\\ \frac{\sqrt{3}}{2} & \frac{1}{2}\end{bmatrix}$ (b) $\begin{bmatrix}\frac{\sqrt{3}}{2} & -\frac{1}{2}\\ \frac{1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}$

(c) $\begin{bmatrix}\frac{\sqrt{3}}{2} & \frac{1}{2}\\ -\frac{1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}$ (d) $\begin{bmatrix}\frac{1}{2} & \frac{\sqrt{3}}{2}\\ -\frac{\sqrt{3}}{2} & \frac{1}{2}\end{bmatrix}$

57. Let A be a matrix such that $A\cdot\begin{bmatrix}1 & 2\\ 0 & 3\end{bmatrix}$ is a scalar matrix and $|3A|=108$. Then A^2 equals **[Online April 15, 2018]**

(a) $\begin{bmatrix}4 & -32\\ 0 & 36\end{bmatrix}$ (b) $\begin{bmatrix}4 & 0\\ -32 & 36\end{bmatrix}$

(c) $\begin{bmatrix}36 & 0\\ -32 & 4\end{bmatrix}$ (d) $\begin{bmatrix}36 & -32\\ 0 & 4\end{bmatrix}$

58. Suppose A is any 3×3 non-singular matrix and $(A-3I)(A-5I)=O$, where $I=I_3$ and $O=O_3$. If $\alpha A+\beta A^{-1}=4I$, then $\alpha+\beta$ is equal to **[Online April 15, 2018]**

(a) 8 (b) 12 (c) 13 (d) 7

59. If $A=\begin{bmatrix}2 & -3\\ -4 & 1\end{bmatrix}$, then adj $(3A^2+12A)$ is equal to : **[2017]**

(a) $\begin{bmatrix}72 & -63\\ -84 & 51\end{bmatrix}$ (b) $\begin{bmatrix}72 & -84\\ -63 & 51\end{bmatrix}$

(c) $\begin{bmatrix}51 & 63\\ 84 & 72\end{bmatrix}$ (d) $\begin{bmatrix}51 & 84\\ 63 & 72\end{bmatrix}$

60. Let A be any 3×3 invertible matrix. Then which one of the following is not always true? **[Online April 8, 2017]**

(a) adj (A) = |A| . A^{-1}
(b) adj (adj(A)) = |A|.A
(c) adj (adj(A)) = $|A|^2$.(adj(A))$^{-1}$
(d) adj (adj(A)) = |A|.(adj(A))$^{-1}$

61. If $A=\begin{bmatrix}5a & -b\\ 3 & 2\end{bmatrix}$ and A adj A = A A^T, then 5a + b is equal to: **[2016]**

(a) 4 (b) 13 (c) –1 (d) 5

62. Let A be a 3×3 matrix such that $A^2-5A+7I=0$.

Statement–I : $A^{-1}=\frac{1}{7}(5I-A)$.

Statement II : the polynomial $A^3-2A^2-3A+\alpha$ can be reduced to $5(A-4I)$. **[Online April 10, 2016]**

Then :
(a) Both the statements are true.
(b) Both the statements are false.
(c) Statement–I is true, but Statement-II is false.
(d) Statement I is false, but Statement-II is true.

63. If A is a 3×3 matrix such that |5.adjA| = 5, then |A| is equal to : **[Online April 11, 2015]**

(a) $\pm\frac{1}{5}$ (b) $\pm\frac{1}{25}$ (c) ±1 (d) ±5

64. If A is an 3×3 non-singular matrix such that $AA'=A'A$ and $B=A^{-1}A'$, then BB' equals: **[2014]**

(a) B^{-1} (b) $(B^{-1})'$ (c) $I+B$ (d) I

65. Let A be a 3×3 matrix such that

$$A\begin{bmatrix}1 & 2 & 3\\ 0 & 2 & 3\\ 0 & 1 & 1\end{bmatrix}=\begin{bmatrix}0 & 0 & 1\\ 1 & 0 & 0\\ 0 & 1 & 0\end{bmatrix}$$

Then A^{-1} is: **[Online April 11, 2014]**

(a) $\begin{bmatrix}3 & 1 & 2\\ 3 & 0 & 2\\ 1 & 0 & 1\end{bmatrix}$ (b) $\begin{bmatrix}3 & 2 & 1\\ 3 & 2 & 0\\ 1 & 1 & 0\end{bmatrix}$

(c) $\begin{bmatrix}0 & 1 & 3\\ 0 & 2 & 3\\ 1 & 1 & 1\end{bmatrix}$ (d) $\begin{bmatrix}1 & 2 & 3\\ 0 & 1 & 1\\ 0 & 2 & 3\end{bmatrix}$

66. If $P=\begin{bmatrix}1 & \alpha & 3\\ 1 & 3 & 3\\ 2 & 4 & 4\end{bmatrix}$ is the adjoint of a 3×3 matrix A and |A| = 4, then α is equal to : **[2013]**

(a) 4 (b) 11 (c) 5 (d) 0

67. Let P and Q be 3×3 matrices $P\neq Q$. If $P^3=Q^3$ and $P^2Q=Q^2P$ then determinant of (P^2+Q^2) is equal to : **[2012]**

(a) – 2 (b) 1
(c) 0 (d) – 1

68. Let $A=\begin{pmatrix}1 & 0 & 0\\ 2 & 1 & 0\\ 3 & 2 & 1\end{pmatrix}$. If u_1 and u_2 are column matrices such that $Au_1=\begin{pmatrix}1\\ 0\\ 0\end{pmatrix}$ and $Au_2=\begin{pmatrix}0\\ 1\\ 0\end{pmatrix}$, then u_1+u_2 is equal to : **[2012]**

(a) $\begin{pmatrix}-1\\ 1\\ 0\end{pmatrix}$ (b) $\begin{pmatrix}-1\\ 1\\ -1\end{pmatrix}$ (c) $\begin{pmatrix}-1\\ -1\\ 0\end{pmatrix}$ (d) $\begin{pmatrix}1\\ -1\\ -1\end{pmatrix}$

69. If A^T denotes the transpose of the matrix $A=\begin{bmatrix}0&0&a\\0&b&c\\d&e&f\end{bmatrix}$, where a, b, c, d, e and f are integers such that $abd \neq 0$, then the number of such matrices for which $A^{-1}=A^T$ is **[Online May 19, 2012]**

(a) 2(3!) (b) 3(2!) (c) 2^3 (d) 3^2

70. Let A and B be real matrices of the form $\begin{bmatrix}\alpha&0\\0&\beta\end{bmatrix}$ and $\begin{bmatrix}0&\gamma\\\delta&0\end{bmatrix}$, respectively. **[Online May 12, 2012]**

Statement 1: $AB - BA$ is always an invertible matrix.

Statement 2: $AB - BA$ is never an identity matrix.

(a) Statement 1 is true, Statement 2 is false.

(b) Statement 1 is false, Statement 2 is true.

(c) Statement 1 is true, Statement 2 is true; Statement 2 is a correct explanation of Statement 1.

(d) Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation of Statement 1.

71. Consider the following relation R on the set of real square matrices of order 3. **[2011RS]**

$R=\{(A,B)\mid A=P^{-1}BP$ for some invertible matrix $P\}$

Statement-1 : R is equivalence relation.

Statement-2 : For any two invertible 3×3 matrices M and N, $(MN)^{-1}=N^{-1}M^{-1}$.

(a) Statement-1 is true, statement-2 is true and statement-2 is a correct explanation for statement-1.

(b) Statement-1 is true, statement-2 is true; statement-2 is not a correct explanation for statement-1.

(c) Statement-1 is true, stement-2 is false.

(d) Statement-1 is false, statement-2 is true.

72. Let A be a 2×2 matrix

Statement -1 : adj (adj A) = A

Statement -2 : |adj A| = |A| **[2009]**

(a) Statement-1 is true, Statement-2 is true. Statement-2 is not a correct explanation for Statement-1.

(b) Statement-1 is true, Statement-2 is false.

(c) Statement -1 is false, Statement-2 is true.

(d) Statement-1 is true, Statement -2 is true. Statement-2 is a correct explanation for Statement-1.

73. Let A be a square matrix all of whose entries are integers. Then which one of the following is true? **[2008]**

(a) If det $A=\pm1$, then A^{-1} exists but all its entries are not necessarily integers

(b) If det $A\neq\pm1$, then A^{-1} exists and all its entries are non integers

(c) If det $A=\pm1$, then A^{-1} exists but all its entries are integers

(d) If det $A=\pm1$, then A^{-1} need not exists

74. If $A^2-A+I=0$, then the inverse of A is **[2005]**

(a) $A+I$ (b) A (c) $A-I$ (d) $I-A$

75. Let $A=\begin{pmatrix}1&-1&1\\2&1&-3\\1&1&1\end{pmatrix}$. and $B=\begin{pmatrix}4&2&2\\-5&0&\alpha\\1&-2&3\end{pmatrix}$. If B is the inverse of matrix A, then α is **[2004]**

(a) 5 (b) –1 (c) 2 (d) –2

76. Let $A=\begin{pmatrix}0&0&-1\\0&-1&0\\-1&0&0\end{pmatrix}$. The only correct statement about the matrix A is **[2004]**

(a) $A^2=I$

(b) $A=(-1)I$, where I is a unit matrix

(c) A^{-1} does not exist

(d) A is a zero matrix

TOPIC 4 Solution of System of Linear Equations

77. The values of λ and μ for which the system of linear equations **[Sep. 06, 2020 (I)]**

$x+y+z=2$

$x+2y+3z=5$

$x+3y+\lambda z=\mu$

has infinitely many solutions are, respectively :

(a) 6 and 8 (b) 5 and 7

(c) 5 and 8 (d) 4 and 9

78. The sum of distinct values of λ for whcih the system of equations

$(\lambda-1)x+(3\lambda+1)y+2\lambda z=0$

$(\lambda-1)x+(4\lambda-2)y+(\lambda+3)z=0$

$2x+(3\lambda+1)y+3(\lambda-1)z=0,$

has non-zero solutions, is ______. **[NA Sep. 06, 2020 (II)]**

79. Let $\lambda\in R$. The system of linear equations

$2x_1-4x_2+\lambda x_3=1$ **[Sep. 05, 2020 (I)]**

$x_1-6x_2+x_3=2$

$\lambda x_1-10x_2+4x_3=3$

(a) exactly one negative value of λ

(b) exactly one positive value of λ

(c) every value of λ

(d) exactly two value of λ

80. If the system of linear equations

$x+y+3z=0$

$x+3y+k^2z=0$

$3x+y+3z=0$

has a non-zero solution (x, y, z) for some $k \in \mathbf{R}$, then $x+\left(\frac{y}{z}\right)$ is equal to : **[Sep. 05, 2020 (II)]**

(a) -3 (b) 9 (c) 3 (d) -9

81. If the system of equations $x-2y+3z=9$, $2x+y+z=b$, $x-7y+az=24$, has infinitely many solutions, then $a-b$ is equal to __________. **[NA Sep. 04, 2020 (I)]**

82. Suppose the vectors x_1, x_2 and x_3 are the solutions of the system of linear equations, $Ax = b$ when the vector b on the right side is equal to b_1, b_2 and b_3 respectively. If

$x_1=\begin{bmatrix}1\\1\\1\end{bmatrix}, x_2=\begin{bmatrix}0\\2\\1\end{bmatrix}, x_3=\begin{bmatrix}0\\0\\1\end{bmatrix}, b_1=\begin{bmatrix}1\\0\\0\end{bmatrix}, b_2=\begin{bmatrix}0\\2\\0\end{bmatrix}$ and

$b_3=\begin{bmatrix}0\\0\\2\end{bmatrix}$, then the determinant of A is equal to :

[Sep. 04, 2020 (II)]

(a) 4 (b) 2

(c) $\frac{1}{2}$ (d) $\frac{3}{2}$

83. If the system of equations

$x+y+z=2$

$2x+4y-z=6$

$3x+2y+\lambda z=\mu$

has infinitely many solutions, then : **[Sep. 04, 2020 (II)]**

(a) $\lambda+2\mu=14$ (b) $2\lambda-\mu=5$

(c) $\lambda-2\mu=-5$ (d) $2\lambda+\mu=14$

84. Let S be the set of all integer solutions, (x, y, z), of the system of equations

$x-2y+5z=0$

$-2x+4y+z=0$

$-7x+14y+9z=0$

such that $15 \le x^2+y^2+z^2 \le 150$. Then, the number of elements in the set S is equal to __________.

[NA Sep. 03, 2020 (II)]

85. Let S be the set of all $\lambda \in \mathbf{R}$ for which the system of linear equations **[Sep. 02, 2020 (I)]**

$2x-y+2z=2$

$x-2y+\lambda z=-4$

$x+\lambda y+z=4$

has no solution. Then the set S

(a) contains more than two elements.

(b) is an empty set.

(c) is a singleton.

(d) contains exactly two elements.

86. Let $A = \{X=(x, y, z)^T : PX = 0 \text{ and } x^2+y^2+z^2=1\}$,

where $P=\begin{bmatrix}1 & 2 & 1\\-2 & 3 & -4\\1 & 9 & -1\end{bmatrix}$, then the set A :

[Sep. 02, 2020 (II)]

(a) is a singleton

(b) is an empty set

(c) contains more than two elements

(d) contains exactly two elements

87. The following system of linear equations

$7x+6y-2z=0$

$3x+4y+2z=0$

$x-2y-6z=0$, has **[Jan. 9, 2020 (II)]**

(a) infinitely many solutions, (x, y, z) satisfying $y=2z$.

(b) no solution.

(c) infinitely many solutions, (x, y, z) satisfying $x=2z$.

(d) only the trivial solution.

88. For which of the following ordered pairs (μ, δ), the system of linear equations

$x+2y+3z=1$

$3x+4y+5z=\mu$

$4x+4y+4z=\delta$

is inconsistent? **[Jan. 8, 2020 (I)]**

(a) (4, 3) (b) (4, 6)

(c) (1, 0) (d) (3, 4)

89. The system of linear equations

$\lambda x+2y+2z=5$

$2\lambda x+3y+5z=8$

$4x+\lambda y+6z=10$ has: **[Jan. 8, 2020 (II)]**

(a) no solution when $\lambda=8$

(b) a unique solution when $\lambda=-8$

(c) no solution when $\lambda=2$

(d) infinitely many solutions when $\lambda=2$

90. If the system of linear equations

$2x + 2ay + az = 0$

$2x + 3by + bz = 0$

$2x + 4cy + cz = 0,$

where $a, b, c \in \mathbf{R}$ are non-zero and distinct; has a non-zero solution, then: **[Jan. 7, 2020 (I)]**

(a) $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

(b) a, b, c are in G.P.

(c) $a + b + c = 0$

(d) a, b, c are in A.P.

91. If the system of linear equations,

$x + y + z = 6$

$x + 2y + 3z = 10$

$3x + 2y + \lambda z = \mu$

has more than two solutions, then $\mu - \lambda^2$ is equal to __________. **[NA Jan. 7, 2020 (II)]**

92. If the system of linear equations

$x + y + z = 5$

$x + 2y + 2z = 6$

$x + 3y + \lambda z = \mu$, $(\lambda, \mu \in \mathbf{R})$, has infinitely many solutions, then the value of $\lambda + \mu$ is : **[April 10, 2019 (I)]**

(a) 12 (b) 9 (c) 7 (d) 10

93. Let λ be a real number for which the system of linear equations:

$x + y + z = 6$

$4x + \lambda y - \lambda z = \lambda - 2$

$3x + 2y - 4z = -5$

has infinitely many solutions. Then λ is a root of the quadratic equation : **[April 10, 2019 (II)]**

(a) $\lambda^2 + 3\lambda - 4 = 0$ (b) $\lambda^2 - 3\lambda - 4 = 0$

(c) $\lambda^2 + \lambda - 6 = 0$ (d) $\lambda^2 - \lambda - 6 = 0$

94. If the system of equations $2x + 3y - z = 0$, $x + ky - 2z = 0$ and $2x - y + z = 0$ has a non-trivial solution (x, y, z), then $\frac{x}{y} + \frac{y}{z} + \frac{z}{x} + k$ is equal to: **[April 09, 2019 (II)]**

(a) $\frac{3}{4}$ (b) $\frac{1}{2}$ (c) $-\frac{1}{4}$ (d) -4

95. The greatest value of $c \in \mathrm{R}$ for which the system of linear equations

$x - cy - cz = 0$; $cx - y + cz = 0$; $cx + cy - z = 0$

has a non-trivial solution, is : **[April 08, 2019 (I)]**

(a) -1 (b) $\frac{1}{2}$ (c) 2 (d) 0

96. If the system of linear equations

$x - 2y + kz = 1$

$2x + y + z = 2$

$3x - y - kz = 3$

has a solution (x, y, z), $z \neq 0$, then (x, y) lies on the straight line whose equation is : **[April 08, 2019 (II)]**

(a) $3x - 4y - 1 = 0$ (b) $4x - 3y - 4 = 0$

(c) $4x - 3y - 1 = 0$ (d) $3x - 4y - 4 = 0$

97. An ordered pair (α, β) for which the system of linear equations

$(1 + \alpha)x + \beta y + z = 2$

$ax + (1 + \beta)y + z = 3$

$\alpha x + \beta y + 2z = 2$

has a unique solution, is : **[Jan. 12, 2019 (I)]**

(a) $(2, 4)$ (b) $(-3, 1)$

(c) $(-4, 2)$ (d) $(1, -3)$

98. The set of all values of λ for which the system of linear equations

$x - 2y - 2z = \lambda x$

$x + 2y + z = \lambda y$

$-x - y = \lambda 2$

has a non-trivial solution : **[Jan. 12, 2019 (II)]**

(a) is a singleton

(b) contains exactly two elements

(c) is an empty set

(d) contains more than two elements

99. If the system of linear equations

$2x + 2y + 3z = a$

$3x - y + 5z = b$

$x - 3y + 2z = c$

where, a, b, c are non-zero real numbers, has more than one solution, then : **[Jan. 11, 2019 (I)]**

(a) $b - c + a = 0$ (b) $b - c - a = 0$

(c) $a + b + c = 0$ (d) $b + c - a = 0$

100. The number of values of $\theta \in (0, \pi)$ for which the system of linear equations

$x + 3y + 7z = 0$

$-x + 4y + 7z = 0$

$(\sin 3\theta)x + (\cos 2\theta)y + 2z = 0$

has a non-trivial solution, is: **[Jan. 10, 2019 (II)]**

(a) three (b) two

(c) four (d) one

101. If the system of equations **[Jan 10, 2019 (I)]**

$x+y+z=5$

$x+2y+3z=9$

$x+3y+\alpha z=\beta$

has infinitely many solutions, then $\beta-\alpha$ equals:

(a) 21 (b) 8 (c) 18 (d) 5

102. If the system of linear equations

$x-4y+7z=g$

$3y-5z=h$

$-2x+5y-9z=k$

is consistent, then : **[Jan. 09, 2019 (II)]**

(a) $g+2h+k=0$

(b) $g+h+2k=0$

(c) $2g+h+k=0$

(d) $g+h+k=0$

103. If the system of linear equations

$x+ky+3z=0$

$3x+ky-2z=0$

$2x+4y-3z=0$

has a non-zero solution (x, y, z), then $\frac{xz}{y^2}$ is equal to : **[2018]**

(a) 10 (b) -30 (c) 30 (d) -10

104. The number of values of k for which the system of linear equations, $(k+2)x+10y=k$, $kx+(k+3)y=k-1$ has no solution, is **[Online April 16, 2018]**

(a) Infinitely many (b) 3

(c) 1 (d) 2

105. Let S be the set of all real values of k for which the system of linear equations

$x+y+z=2$

$2x+y-z=3$

$3x+2y+kz=4$

has a unique solution. Then S is **[Online April 15, 2018]**

(a) an empty set (b) equal to $R-\{0\}$

(c) equal to $\{0\}$ (d) equal to R

106. If the system of linear equations

$x+ay+z=3$

$x+2y+2z=6$

$x+5y+3z=b$

has no solution, then **[Online April 15, 2018]**

(a) $a=1, b\neq 9$ (b) $a\neq -1, b=9$

(c) $a=-1, b=9$ (d) $a=-1, b\neq 9$

107. If S is the set of distinct values of 'b' for which the following system of linear equations **[2017]**

$x+y+z=1$

$x+ay+z=1$

$ax+by+z=0$

has no solution, then S is :

(a) a singleton

(b) an empty set

(c) an infinite set

(d) a finite set containing two or more elements

108. The number of real values of λ for which the system of linear equations

$2x+4y-\lambda z=0$

$4x+\lambda y+2z=0$

$\lambda x+2y+2z=0$

has infinitely many solutions, is : **[Online April 8, 2017]**

(a) 0 (b) 1 (c) 2 (d) 3

109. The system of linear equations

$x+\lambda y-z=0$

$\lambda x-y-z=0$

$x+y-\lambda z=0$

has a non-trivial solution for: **[2016]**

(a) exactly two values of λ.

(b) exactly three values of λ.

(c) infinitely many values of λ.

(d) exactly one value of λ.

110. The set of all values of λ for which the system of linear equations : **[2015]**

$2x_1-2x_2+x_3=\lambda x_1$

$2x_1-3x_2+2x_3=\lambda x_2$

$-x_1+2x_2=\lambda x_3$

has a non-trivial solution,

(a) contains two elements.

(b) contains more than two elements

(c) is an empty set.

(d) is a singleton

111. If a, b, c are non-zero real numbers and if the system of equations **[Online April 9, 2014]**

$(a-1)x=y+z,$

$(b-1)y=z+x,$

$(c-1)z=x+y,$

has a non-trivial solution, then ab + bc + ca equals:

(a) $a+b+c$ (b) abc

(c) 1 (d) -1

112. The number of values of k, for which the system of equations:

$(k+1)x+8y=4k$

$kx+(k+3)y=3k-1$

has no solution, is **[2013]**

(a) infinite (b) 1

(c) 2 (d) 3

113. Consider the system of equations :

$x+ay=0$, $y+az=0$ and $z+ax=0$. Then the set of all real values of 'a' for which the system has a unique solution is: **[Online April 25, 2013]**

(a) $R-\{1\}$ (b) $R-\{-1\}$

(c) $\{1,-1\}$ (d) $\{1,0,-1\}$

114. **Statement-1:** The system of linear equations

$x + (\sin\alpha)y + (\cos\alpha)z = 0$
$x + (\cos\alpha)y + (\sin\alpha)z = 0$
$x - (\sin\alpha)y - (\cos\alpha)z = 0$

has a non-trivial solution for only one value of α lying in the interval $\left(0, \frac{\pi}{2}\right)$.

Statement-2: The equation in α

$$\begin{vmatrix} \cos\alpha & \sin\alpha & \cos\alpha \\ \sin\alpha & \cos\alpha & \sin\alpha \\ \cos\alpha & -\sin\alpha & -\cos\alpha \end{vmatrix} = 0$$

has only one solution lying in the interval $\left(0, \frac{\pi}{2}\right)$.

[Online April 23, 2013]

(a) Statement-1 is true, Statement-2 is true, Statement-2 is **not** correct explantion for Statement-1.
(b) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explantion for Statement-1.
(c) Statement-1 is true, Statement-2 is false.
(d) Statememt-1 is false, Statement-2 is true.

115. If the system of linear equations :

$x_1 + 2x_2 + 3x_3 = 6$
$x_1 + 3x_2 + 5x_3 = 9$
$2x_1 + 5x_2 + ax_3 = b$

is consistent and has infinite number of solutions, then : **[Online April 22, 2013]**

(a) $a = 8$, b can be any real number
(b) $b = 15$, a can be any real number
(c) $a \in R - \{8\}$ and $b \in R - \{15\}$
(d) $a = 8, b = 15$

116. **Statement 1:** If the system of equations $x + ky + 3z = 0$, $3x + ky - 2z = 0$, $2x + 3y - 4z = 0$ has a non-trivial solution, then the value of k is $\frac{31}{2}$.

Statement 2: A system of three homogeneous equations in three variables has a non trivial solution if the determinant of the coefficient matrix is zero. **[Online May 26, 2012]**

(a) Statement 1 is false, Statement 2 is true.
(b) Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1.
(c) Statement 1 is true, Statement 2 is true, , Statement 2 is not a correct explanation for Statement 1.
(d) Statement 1 is true, Statement 2 is false.

117. If the system of equations **[Online May 7, 2012]**

$x + y + z = 6$
$x + 2y + 3z = 10$
$x + 2y + \lambda z = 0$

has a unique solution, then λ is not equal to

(a) 1 (b) 0 (c) 2 (d) 3

118. If the trivial solution is the only solution of the system of equations **[2011RS]**

$x - ky + z = 0$
$kx + 3y - kz = 0$
$3x + y - z = 0$

then the set of all values of k is :

(a) $R - \{2, -3\}$ (b) $R - \{2\}$
(c) $R - \{-3\}$ (d) $\{2, -3\}$

119. The number of values of k for which the linear equations $4x + ky + 2z = 0$, $kx + 4y + z = 0$ and $2x + 2y + z = 0$ possess a non-zero solution is **[2011]**

(a) 2 (b) 1 (c) zero (d) 3

120. Consider the system of linear equations; **[2010]**

$x_1 + 2x_2 + x_3 = 3$
$2x_1 + 3x_2 + x_3 = 3$
$3x_1 + 5x_2 + 2x_3 = 1$

The system has

(a) exactly 3 solutions
(b) a unique solution
(c) no solution
(d) infinite number of solutions

121. Let a, b, c be any real numbers. Suppose that there are real numbers x, y, z not all zero such that $x = cy + bz$, $y = az + cx$, and $z = bx + ay$. Then $a^2 + b^2 + c^2 + 2abc$ is equal to

(a) 2 (b) –1 **[2008]**
(c) 0 (d) 1

122. The system of equations

$\alpha x + y + z = \alpha - 1$
$x + \alpha y + z = \alpha - 1$
$x + y + \alpha z = \alpha - 1$

has infinite solutions, if α is **[2005]**

(a) -2 (b) either -2 or 1
(c) not -2 (d) 1

123. If the system of linear equations **[2003]**

$x + 2ay + az = 0$; $x + 3by + bz = 0$;
$x + 4cy + cz = 0$ has a non - zero solution, then a, b, c.

(a) satisfy $a + 2b + 3c = 0$
(b) are in A.P
(c) are in G.P
(d) are in H.P.

Hints & Solutions

1. **(d)** $\because A=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$

$\therefore A^n=\begin{bmatrix}\cos n\theta & \sin n\theta\\ -\sin n\theta & \cos n\theta\end{bmatrix}, n\in\mathbf{N}$

$\therefore B=A+A^4$

$=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}+\begin{bmatrix}\cos 4\theta & \sin 4\theta\\ -\sin 4\theta & \cos 4\theta\end{bmatrix}$

$\therefore B=\begin{bmatrix}\cos\frac{\pi}{5}+\cos\frac{4\pi}{5} & \sin\frac{\pi}{5}+\sin\frac{4\pi}{5}\\ -\sin\frac{\pi}{5}-\sin\frac{4\pi}{5} & \cos\frac{\pi}{5}+\cos\frac{4\pi}{5}\end{bmatrix}$

Then, $\det(B)=2\sin\left(\frac{\pi}{5}\right)\cdot\begin{vmatrix}0 & 1\\ -1 & 0\end{vmatrix}$

$=\frac{\sqrt{10-2\sqrt{5}}}{2}\approx\frac{2.35}{2}\approx 1.175$

$\therefore \det B\in(1,2)$

2. **(c)** $\Delta=\begin{vmatrix}x-2 & 2x-3 & 3x-4\\ 2x-3 & 3x-4 & 4x-5\\ 3x-5 & 5x-8 & 10x-17\end{vmatrix}$

$\Rightarrow\Delta=\begin{vmatrix}x-2 & x-1 & x-1\\ 2x-3 & x-1 & x-1\\ 3x-5 & 2x-3 & 5x-9\end{vmatrix}$ $\begin{bmatrix}C_3\to C_3-C_2\\ C_2\to C_2-C_1\end{bmatrix}$

$\Rightarrow\Delta=\begin{vmatrix}x-2 & x-1 & x-1\\ x-1 & 0 & 0\\ 3x-5 & 2x-3 & 5x-9\end{vmatrix}$ $[R_2\to R_2-R_1]$

$\Rightarrow\Delta=-(x-1)[(x-1)(5x-9)-(x-1)(2x-3)]$

$\Rightarrow\Delta=-(x-1)[(5x^2-14x+9)-(2x^2-5x+3)]$

$=-3x^3+12x^2-15x+6$

So, $B+C=-3$

3. **(d)** If $f(x)=\begin{vmatrix}x+a & x+2 & x+1\\ x+b & x+3 & x+2\\ x+c & x+4 & x+3\end{vmatrix}$

$R_1=R_1+R_3-2R_2$

$\Rightarrow f(x)=\begin{vmatrix}1 & 0 & 0\\ x+b & x+3 & x+2\\ x+c & x+4 & x+3\end{vmatrix}$

$\Rightarrow f(x)=1 \Rightarrow f(50)=1$

4. **(d)** $\Delta_1=\begin{vmatrix}x & \sin\theta & \cos\theta\\ -\sin\theta & -x & 1\\ \cos\theta & 1 & x\end{vmatrix}$

$=(x-x^2-1)-\sin\theta(-x\sin\theta-\cos\theta)$
$+\cos\theta(-\sin\theta+x\cos\theta)$

$=-x^3-x+x\sin^2\theta+\sin\theta\cos\theta-\cos\theta\sin\theta+x\cos^2\theta$

$=-x^3-x+x=-x^3$

Similarly, $\Delta_2=-x^3$ Then, $\Delta_1+\Delta_2=-2x^3$

5. **(b)** Given $\begin{vmatrix}x & -6 & -1\\ 2 & -3x & x-3\\ -3 & 2x & x+2\end{vmatrix}=0$

On expanding,

$x(-3x^2-6x-2x^2+6x)-6(-3x+9-2x-4)$
$-(4x-9x)=0$

$\Rightarrow x(-5x^2)-6(-5x+5)-4x+9x=0$

$\Rightarrow x^3-7x+6=0$

$\because$ all the roots are real.

$\therefore$ sum of real roots $=\frac{0}{1}=0$

6. **(a)** $|A|=\begin{vmatrix}2 & b & 1\\ b & b^2+1 & b\\ 1 & b & 2\end{vmatrix}$

$=2(2b^2+2-b^2)-b(2b-b)+1(b^2-b^2-1)$

$=2b^2+4-b^2-1=b^2+3$

$\frac{|A|}{b}=b+\frac{3}{b}$

$\because \frac{b+\frac{3}{b}}{2}\geq\left(b\frac{3}{b}\right)^{\frac{1}{2}}\Rightarrow b+\frac{3}{b}\geq 2\sqrt{3}$

$\therefore \frac{|A|}{b}\geq 2\sqrt{3}$

Minimum value of $\frac{|A|}{b}$ is $2\sqrt{3}$.

7. (b) Here, $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A+Bx)(x-A)^2$

Put $x=0 \Rightarrow \begin{vmatrix} -4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4 \end{vmatrix} = A^3 \Rightarrow A^3 = (-4)^3$

$\Rightarrow A = -4$

$\Rightarrow \begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (Bx-4)(x+4)^2$

Now take x common from both the sides

$\therefore \begin{vmatrix} 1-\frac{4}{x} & 2x & 2x \\ 2x & 1-\frac{4}{x} & 2x \\ 2x & 2x & 1-\frac{4}{x} \end{vmatrix} = (B-\frac{4}{x})(1+\frac{4}{x})^2$

Now take $x \to \infty$, then $\frac{1}{x} \to 0$

$\Rightarrow \begin{vmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix} = B \Rightarrow B = 5$

$\therefore$ ordered pair (A, B) is (–4, 5)

8. (c) Since the given determinant is equal to zero.

$\Rightarrow$ $0(0-\cos x \sin x) - \cos x(0-\cos^2 x) - \sin x (\sin^2 x - 0) = 0$

$\Rightarrow$ $\cos^3 x - \sin^3 x = 0$

$\Rightarrow$ $\tan^3 = 1 \Rightarrow \tan x = 1$

$\therefore \sum_{x\in s} \tan\left(\frac{\pi}{3}+x\right) = \sum_{x\in s} \frac{\tan \pi/3 + \tan x}{1-\tan \pi/3 . \tan x}$

$\sum_{x\in s} \frac{\sqrt{3}+1}{1-\sqrt{3}}$

$\sum_{x\in s} \frac{\sqrt{3}+1}{1-\sqrt{3}} \times \frac{1+\sqrt{3}}{1+\sqrt{3}} \Rightarrow \sum_{x\in s} \frac{1+3+2\sqrt{3}}{-2}$

$= -2-\sqrt{3}$

9. (d) $A = \begin{bmatrix} -4 & -1 \\ 3 & 1 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} -4 & -1 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} -4 & -1 \\ 3 & 1 \end{bmatrix}$

$= \begin{bmatrix} 13 & 3 \\ -9 & -2 \end{bmatrix}$ and $|A| = 1$.

Now, $A^{2016} - 2A^{2015} - A^{2014} = A^{2014}(A^2 - 2A - I)$

$\Rightarrow |A^{2016} - 2A^{2015} - A^{2014}| = |A^{2014}||A^2 - 2A - I|$

$= |A|^{2014} \begin{vmatrix} 20 & 5 \\ -15 & -5 \end{vmatrix} = -25$

10. (a) Let $\begin{vmatrix} x^2+x & x+1 & x-2 \\ 2x^2+3x-1 & 3x & 3x-3 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix} = ax-12$

Put $x=-1$, we get

$\begin{vmatrix} 0 & 0 & -3 \\ -2 & -3 & 0 \\ 2 & -3 & -3 \end{vmatrix} = -a-12$

$\Rightarrow -3(6+6) = -a-12 \Rightarrow -36+12 = a$

$\Rightarrow a = 24$

11. (d) $\begin{vmatrix} x & 1 & 1 \\ 1 & y & 1 \\ 1 & 1 & z \end{vmatrix} \geq 0$

$xyz - x - y - z + 2 \geq 0$

$xyz + 2 \geq x + y + z \geq 3(xyz)^{1/3}$

$xyz + 2 - 3(xyz)^{1/3} \geq 0$

$ut(xyz) = t^3$

$t^3 - 3t + 2 \geq 0$

$(t+2)(t-1)^2 \geq 0$

$[t = -2]\ t^3 = -8$

12. (c) Let $f(\theta) = \begin{vmatrix} 1 & \cos\theta & 1 \\ -\sin\theta & 1 & -\cos\theta \\ -1 & \sin\theta & 1 \end{vmatrix}$

$= (1+\sin\theta\cos\theta) - \cos\theta(-\sin\theta - \cos\theta) + 1(-\sin^2\theta + 1)$

$= 1 + \sin\theta\cos\theta + \sin\theta\cos\theta + \cos^2\theta - \sin^2\theta + 1$

$= 2 + 2\sin\theta\cos\theta + \cos 2\theta$

$= 2 + \sin 2\theta + \cos 2\theta$ ...(1)

Now, maximum value of (1)

is $2 + \sqrt{1^2+1^2} = 2+\sqrt{2}$

and minimum value of (1) is

$2 - \sqrt{1^2+1^2} = 2-\sqrt{2}$.

13. (a) det $[(I+B)^{50} - 50B]$

$= \det[{}^{50}C_0 I + {}^{50}C_1 B + {}^{50}C_2 B^2 + {}^{50}C_3 B^3 + \ldots + {}^{50}C_{50} B^{50} B^{50} - 50B]$

{All terms having B^n, $2 \leq n \leq 50$ will be zero because given that $B^2 = 0$}

$= \det[I + 50B - 50B] = \det[I] = 1$

14. (d) The matrices in the form

$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, a_{ij} \in \{0, 1, 2\}, a_{11} = a_{12}$ are

$$\begin{bmatrix} 0 & 0/1/2 \\ 0/1/2 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0/1/2 \\ 0/1/2 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 0/1/2 \\ 0/1/2 & 2 \end{bmatrix}$$

At any place, 0/1/2 means 0, 1 or 2 will be the element at that place.

Hence there are total $27 = 3 \times 3 + 3 \times 3 + 3 \times 3)$ matrices of the above form. Out of which the matrices which are singular are

$$\begin{bmatrix} 0 & 0/1/2 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1/2 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

Hence there are total $7(= 3 + 2 + 1 + 1)$ singular matrices.

Therefore number of all non-singular matrices in the given form $= 27 - 7 = 20$

15. (b) $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$\begin{bmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$b(a + d) = 0, b = 0$ or $a = -d$ ...(1)

$c(a + d) = 0, c = 0$ or $a = -d$ ...(2)

$a^2 + bc = 1, bc + d^2 = 1$ ...(3)

'a' and 'd' are diagonal elements $a + d = 0$ statement-1 is correct.

Now, $\det(A) = ad - bc$

Now, from (3) $a^2 + bc = 1$ and $d^2 + bc = 1$

So, $a^2 - d^2 = 0$

Adding $a^2 + d^2 + 2bc = 2$

$\Rightarrow (a + d)^2 - 2ad + 2bc = 2$

or $0 - 2(ad - bc) = 2$

So, $ad - bc = 1 \Rightarrow \det(A) = -1$

So, statement – 2 is also true.

But statement – 2 is not the correct explanation of statement-1.

16. (d) We know that determinant of skew symmetric matrix of odd order is zero.

So, statement-1 is true.

We know that det $\left(A^T\right) =$ det (A).

det (– A) = – (– 1)^n det (A).

where A is a $n \times n$ order matrix.

So, statement-2 is false.

17. (b) Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ where a, b, c, d $\neq 0$

$$A^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = I$$

$$\Rightarrow A^2 = \begin{bmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\Rightarrow a^2 + bc = 1, bc + d^2 = 1$

$ab + bd = ac + cd = 0$

$c \neq 0$ and $b \neq 0 \Rightarrow a + d = 0$

$|A| = ad - bc = -a^2 - bc = -1$

Also if A $\neq$ I, then tr(A) $= a + d = 0$.

$\therefore$ Statement-1 true and statement-2 false.

18. (d) Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Given that $A^2 =$ I

$$\Rightarrow \begin{bmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\Rightarrow$ $a^2 + bc = 1$ and $ab + bd = 0$

$ac + cd = 0$ and $bc + d^2 = 1$

From these four equations,

$a^2 + bc = bc + d^2 \Rightarrow a^2 = d^2$

and $b(a + d) = 0 = c(a + d) \Rightarrow a = -d$

$|A| = ad - bc = -a^2 - bc = -1$

Also if $A \neq I$ then $tr(A) = a + d = 0$

$\therefore$ Statement 2 is false.

19. (a) Given that $A = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$ and $|A^2| = 25$

$$\therefore A^2 = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}\begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 25 & 25\alpha + 5\alpha^2 & 5\alpha + 25\alpha^2 + 5\alpha \\ 0 & \alpha^2 & 5\alpha^2 + 25\alpha \\ 0 & 0 & 25 \end{bmatrix}$$

$\therefore |A^2| = 25\ (25\alpha^2)$

$\therefore 25 = 25\ (25\alpha^2) \Rightarrow |\alpha| = \frac{1}{5}$

20. (b) $\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix}$

Expand through R_1

$= 1\left(\omega^{3n}-1\right)-\omega^{n}\left(\omega^{2n}-\omega^{2n}\right)+\omega^{2n}\left(\omega^{n}-\omega^{4n}\right)$

$= \omega^{3n}-1-0+\omega^{3n}-\omega^{6n}$

$= 1-1+1-1=0\left[\because \omega^{3n}=1\right]$

21. **(b)** Applying $C_2 \to C_2 - C_1$

$$f(\theta)=\begin{vmatrix} -\sin^2\theta & -1 & 1 \\ -\cos^2\theta & -1 & 1 \\ 12 & -2 & -2 \end{vmatrix}$$

$= 4(\cos^2\theta-\sin^2\theta)$

$= 4\cos 2\theta,\ \theta \in \left(\frac{\pi}{4},\frac{\pi}{2}\right)$

Max. $f(\theta)=M=0$

Min. $f(\theta)=m=-4$

So, $(m, M)=(-4, 0)$

22. **(b)** Use properties of determinant

$$\begin{vmatrix} x & a+y & x+a \\ y & b+y & y+b \\ z & c+y & z+c \end{vmatrix}=\begin{vmatrix} x & a & x+a \\ y & b & y+b \\ z & c & z+c \end{vmatrix}+y\begin{vmatrix} x & 1 & x+a \\ y & 1 & y+b \\ z & 1 & z+c \end{vmatrix}$$

$$=0+y\begin{vmatrix} x & 1 & x+a \\ y-x & 0 & 0 \\ z-x & 0 & -1 \end{vmatrix} \quad \begin{bmatrix} R_2 \to R_2-R_1, \\ R_3 \to R_3-R_1 \end{bmatrix}$$

$= -y(x-y)=-y(b-a)=y(a-b)$

23. **(b)** $D=\frac{1}{2}\begin{vmatrix} 0 & 2 & 1 \\ 1 & -1 & 1 \\ x' & y' & 1 \end{vmatrix}=5$

$\Rightarrow\ -2(1-x')+(y'+x')=\pm 10$

$\Rightarrow\ -2+2x'+y'+x'=\pm 10$

$\Rightarrow\ 3x'+y'=12$ or $3x'+y'=-8$

$\therefore\ \lambda=3,-2$

24. **(d)** It is given that $|B|=81$

$$\therefore\ |B|=\begin{vmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{vmatrix}=\begin{vmatrix} 3^0 a_{11} & 3^1 a_{12} & 3^2 a_{13} \\ 3^1 a_{21} & 3^2 a_{22} & 3^3 a_{23} \\ 3^2 a_{31} & 3^3 a_{32} & 3^4 a_{33} \end{vmatrix}$$

$\Rightarrow\ 81=3^3\cdot 3^2\cdot 3^1|A|$

$\Rightarrow\ 3^4=3^6|A| \Rightarrow |A|=\frac{1}{9}$

25. **(a)** $C_1 \to C_1 + C_2$

$$\begin{vmatrix} 2 & \sin^2\theta & 4\cos 6\theta \\ 2 & 1+\sin^2\theta & 4\cos 6\theta \\ 1 & \sin^2\theta & 1+4\cos 6\theta \end{vmatrix}=0$$

$R_1 \to R_1-R_2,\ R_2 \to R_2-R_3$

$$\begin{vmatrix} 0 & -1 & 0 \\ 1 & 1 & -1 \\ 1 & \sin^2\theta & (1+4\cos 6\theta) \end{vmatrix}=0$$

On expanding, we get $2+4\cos 6\theta=0$

$\cos 6\theta=-\frac{1}{2} \because \theta \in \left(0,\frac{\pi}{3}\right) \Rightarrow 6\theta\ (0, 2\pi)$

Therefore, $6\theta=\frac{2\pi}{3}$ or $\frac{4\pi}{3} \Rightarrow \theta=\frac{\pi}{9}$ or $\frac{2\pi}{9}$

26. **(c)** Let $\alpha=\omega$ and $\beta=\omega^2$ are roots of $x^2+x+1=0$

$$\&\ Let\ \Delta=\begin{vmatrix} y+1 & \omega & \omega^2 \\ \omega & y+\omega^2 & 1 \\ \omega^2 & 1 & y+\omega \end{vmatrix}=\Delta$$

Applying $C_1 \to C_1+C_2+C_3$

$$\Delta=\begin{vmatrix} y+1+\omega+\omega^2 & \omega & \omega^2 \\ y+1+\omega+\omega^2 & y+\omega^2 & 1 \\ 1+\omega+\omega^2+y & 1 & y+\omega \end{vmatrix}$$

$$\Delta=\begin{vmatrix} y & \omega & \omega^2 \\ y & y+\omega^2 & 1 \\ y & 1 & y+\omega \end{vmatrix}\ (\because 1+\omega+\omega^2=0)$$

$$\Delta=y\begin{vmatrix} 1 & \omega & \omega^2 \\ 1 & y+\omega^2 & 1 \\ 1 & 1 & y+\omega \end{vmatrix}$$

Applying $R_2 \to R_2-R_1\ \&\ R_3 \to R_3-R_1$

$$\Delta=y\begin{vmatrix} y+\omega^2-\omega & 1-\omega^2 \\ 1-\omega & y+\omega-\omega^2 \end{vmatrix}$$

$\Rightarrow \Delta=y\left[y-(\omega-\omega^2)(y+(\omega-\omega^2)-(1-\omega)(1-\omega^2)\right]$

$\Rightarrow \Delta=y\left[y^2-(\omega-\omega^2)^2-1+\omega^2+\omega-\omega^3\right]$

$\Rightarrow \Delta = y\left[y^2 - \omega^2 - \omega^4 + 2\omega^3 - 1 + \omega^2 + \omega^4 - \omega^3\right]$

$(\because \omega^4 = \omega)$

$\Rightarrow \Delta = y(y^2) = y^3$

27. **(c)** Consider, $|A| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & b & c \\ 4 & b^2 & c^2 \end{vmatrix}$

$C_2 \to C_2 - C_1, C_3 \to C_3 - C_1$

$$|A| = \begin{vmatrix} 1 & 0 & 0 \\ 2 & b-2 & c-2 \\ 4 & (b-2)(b+2) & (c-2)(c+2) \end{vmatrix}$$

$= (b-2)(c-2)(c-b)$

$\therefore$ 2, b, c are in A.P.

$\therefore$ $(b-2) = (c-b) = d$ and $c - 2 = 2d$

$\Rightarrow |A| = d.2d.d = 2d^3$

$\because |A| \in [2, 16] \Rightarrow 1 \le d^3 \le 8 \Rightarrow 1 \le d \le 2$

$4 \le 2d + 2 \le 6 \Rightarrow 4 \le c \le 6$

28. **(d)** $|A| = \begin{vmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix}$

$= \begin{vmatrix} 0 & 0 & 2 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix} \quad R_1 \to R_1 + R_3$

$= 2(\sin^2\theta + 1)$

Since, $\theta \in \left(\frac{3\pi}{4}, \frac{5\pi}{4}\right) \Rightarrow \sin^2\theta \in \left(0, \frac{1}{2}\right)$

$\therefore$ $\det(A) \in [2, 3)$

$[2, 3) \subset \left(\frac{3}{2}, 3\right]$

29. **(d)** $\Delta = \begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$

$R_1 \to R_1 + R_2 + R_3$

$$\Delta = \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$$= (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$C_1 \to C_1 - C_3, C_2 \to C_2 - C_3$

$$\Delta = (a+b+c)\begin{vmatrix} 0 & 0 & 1 \\ 0 & -b-c-a & 2b \\ c+a+b & c+a+b & c-a-b \end{vmatrix}$$

$= (a+b+c)(a+b+c)^2$

Hence, $x = -2(a+b+c)$

30. **(a)** $\det(A) = \begin{vmatrix} -2 & 4+d & \sin\theta - 2 \\ 1 & \sin\theta + 2 & d \\ 5 & 2\sin\theta - d & -\sin\theta + 2 + 2d \end{vmatrix}$

Applying $R_3 \to R_3 - 2R_2 + R_1$ we get

$$\det(A) = \begin{vmatrix} -2 & 4+d & \sin\theta - 2 \\ 1 & \sin\theta + 2 & d \\ 1 & 0 & 0 \end{vmatrix}$$

$= \quad d(4+d) - (\sin^2\theta - 4)$

$\Rightarrow \quad \det(A) = d^2 + 4d + 4 - \sin^2\theta = (d+2)^2 - \sin^2\theta$

Minimum value of det (A) is attained when $\sin^2\theta = 1$

$\therefore \quad (d+2)^2 - 1 = 8 \Rightarrow (d+2)^2 = 9 \Rightarrow d + 2 = \pm 3$

$\Rightarrow \quad d = -5$ or 1

31. **(b)** Let common ratio of G.P. be R

$\Rightarrow \quad a_2 = a_1 R, a_3 = a_1 R^2, \ldots a^{10} = a_1 R^9$

$C_1 \to C_1 - C_2, C_2 \to C_2 - C_3$

$$\Delta = \begin{vmatrix} \ln\left(\frac{a_1^r a_2^k}{a_2^r a_3^k}\right) & \ln\left(\frac{a_2^r a_3^k}{a_3^r a_4^k}\right) & \ln a_3^r a_4^k \\ \ln\left(\frac{a_4^r a_5^k}{a_5^r a_6^k}\right) & \ln\left(\frac{a_5^r a_6^k}{a_6^r a_7^k}\right) & \ln a_6^r a_7^k \\ \ln \frac{a_7^r a_8^k}{a_8^r a_9^k} & \ln\left(\frac{a_8^r a_9^k}{a_9^r a_{10}^k}\right) & \ln a_9^r a_{10}^k \end{vmatrix}$$

$$\Delta = \begin{vmatrix} \ln \frac{1}{R^{r+k}} & \ln \frac{1}{R^{r+k}} & \ln a_3^r a_4^k \\ \ln \frac{1}{R^{r+k}} & \ln \frac{1}{R^{r+k}} & \ln a_6^r a_7^k \\ \ln \frac{1}{R^{r+k}} & \ln \frac{1}{R^{r+k}} & \ln a_9^r a_{10}^k \end{vmatrix} = 0$$

$\forall r, K \in N$

Hence, number of elements in S is infinitely many.

32. **(a)** $\det(A) = |A|$

$$= \begin{vmatrix} e^t & e^{-t}\cos t & e^{-t}\sin t \\ e^t & -e^{-t}\cos t - e^{-t}\sin t & -e^{-t}\sin t + e^{-t}\cos t \\ e^t & 2e^{-t}\sin t & -2e^{-t}\cos t \end{vmatrix}$$

$$= e^t \cdot e^{-t} \cdot e^{-t} \begin{vmatrix} 1 & \cos t & \sin t \\ 1 & -\cos t - \sin t & -\sin t + \cos t \\ 1 & 2\sin t & -2\cos t \end{vmatrix}$$

$$= e^{-t} \begin{vmatrix} 0 & 2\cos t + \sin t & 2\sin t - \cos t \\ 0 & -\cos t - 3\sin t & -\sin t + 3\cos t \\ 1 & 2\sin t & -2\cos t \end{vmatrix} \begin{matrix} R_1 \to R_1 - R_2 \\ R_2 \to R_2 + R_3 \end{matrix}$$

$$= e^{-t} \begin{vmatrix} 0 & -5\sin t & 5\cos t \\ 0 & -\cos t - 3\sin t & -\sin t + 3\cos t \\ 1 & 2\sin t & -2\cos t \end{vmatrix} R_1 \to R_1 + 2R_2$$

$= e^{-t}[(-5\sin t)(-\sin t + 3\cos t) - 5\cos t(-\cos t - 3\sin t)$

$= 5e^{-t} \neq 0, \forall t \in R$

$\therefore A$ is invertible.

33. **(a)** Let A (k, –3k), B(5, k) and C(–k + 2), we have

$$\frac{1}{2}\left\|\begin{matrix} k & -3k & 1 \\ 5 & k & 1 \\ -k & 2 & 1 \end{matrix}\right\| = 28$$

$\Rightarrow 5k^2 + 13k - 46 = 0$

or $5k^2 + 13k + 66 = 0$

Now, $5k^2 + 13k - 46 = 0$

$\Rightarrow k = \dfrac{-13 \pm \sqrt{1089}}{10} \therefore k = \dfrac{-23}{5}; k = 2$

since k is an integer, $\therefore k = 2$

Also $5k^2 + 13k + 66 = 0$

$\Rightarrow k = \dfrac{-13 \pm \sqrt{-1151}}{10}$

So no real solution exist

A(2, –6), B(5, 2) and C(–2, 2)

For orthocentre H (α, β)

$BH \perp AC$

$\therefore \left(\dfrac{\beta - 2}{\alpha - 5}\right)\left(\dfrac{8}{-4}\right) = -1$

$\Rightarrow \alpha - 2\beta = 1$ ...(1)

Also $CH \perp AB$

$\therefore \left(\dfrac{\beta - 2}{\alpha + 2}\right)\left(\dfrac{8}{3}\right) = -1$

$\Rightarrow 3\alpha + 8\beta = 1$ ... (2)

Solving (1) and (2), we get

$\alpha = 2, \beta = \dfrac{1}{2}$

orthocentre is $\left(2, \dfrac{1}{2}\right)$

34. **(b)** Given $2\omega + 1 = z$;

and $z = \sqrt{3}i \Rightarrow \omega = \dfrac{\sqrt{3}i - 1}{2}$

$\Rightarrow \omega$ is complex cube root of unity

Applying $R_1 \to R_1 + R_2 + R_3$

$$= \begin{vmatrix} 3 & 0 & 0 \\ 1 & -\omega^2 - 1 & \omega^2 \\ 1 & \omega^2 & \omega \end{vmatrix}$$

$= 3(-1 - \omega - \omega) = -3(1 + 2\omega) = -3z$

$\Rightarrow k = -z$

35. **(c)** $\begin{vmatrix} \cos x & \sin x & \sin x \\ \sin x & \cos x & \sin x \\ \sin x & \sin x & \cos x \end{vmatrix} = 0$

$R_1 \to R_1 - R_2$

$R_2 \to R_2 - R_3$

$$\begin{vmatrix} \cos x - \sin x & \sin x - \cos x & 0 \\ 0 & \cos x - \sin x & \sin x - \cos x \\ \sin x & \sin x & \cos x \end{vmatrix} = 0$$

$C_2 \to C_2 + C_3$

$$\begin{vmatrix} \cos x - \sin x & \sin x - \cos x & 0 \\ 0 & 0 & \sin x - \cos x \\ \sin x & \sin x & \cos x \end{vmatrix} = 0$$

Expanding using second row

$2\sin x(\sin x - \cos x)^2 = 0$

$\sin x = 0$ or $\sin x = \cos x$

$x = 0$ or $x = \dfrac{\pi}{4}$

36. **(a)** Consider

$$\begin{vmatrix} 3 & 1 + f(1) & 1 + f(2) \\ 1 + f(1) & 1 + f(2) & 1 + f(3) \\ 1 + f(2) & 1 + f(3) & 1 + f(4) \end{vmatrix}$$

$$= \begin{vmatrix} 1 + 1 + 1 & 1 + \alpha + \beta & 1 + \alpha^2 + \beta^2 \\ 1 + \alpha + \beta & 1 + \alpha^2 + \beta^2 & 1 + \alpha^3 + \beta^3 \\ 1 + \alpha^2 + \beta^2 & 1 + \alpha^3 + \beta^3 & 1 + \alpha^4 + \beta^4 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix} \times \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix} \quad [\because |A| = |A^1|]$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix}^2 = [(1-\alpha)(1-\beta)(\alpha-\beta)]^2$$

So, $K = 1$

37. (d) $\sum_{r=1}^{n-1} r = 1 + 2 + 3 + ... + (n-1) = \frac{n(n-1)}{2}$

$\sum_{r=1}^{n-1} (2r-1) = 1 + 3 + 5 + ... + [2(n-1) - 2]$
$= (n-1)^2$

$\sum_{r=1}^{n-1} (3r-2) = 1 + 4 + 7 + .. + (3n - 3 - 2)$
$= \frac{(n-1)(3n-4)}{2}$

$$\therefore \sum_{r=1}^{n-1} \Delta_r = \begin{vmatrix} \Sigma r & \Sigma(2r-1) & \Sigma(3r-2) \\ \frac{n}{2} & n-1 & a \\ \frac{n(n-1)}{2} & (n-1)^2 & \frac{(n-1)(3n-4)}{2} \end{vmatrix}$$

$\sum_{r=1}^{n-1} \Delta_r$ consists of $(n-1)$ determinants in L.H.S. and in R.H.S every constituent of first row consists of (n – 1) elements and hence it can be splitted into sum of $(n-1)$ determinants.

$$\therefore \sum_{r=1}^{n-1} \Delta_r = \begin{vmatrix} \frac{n(n-1)}{2} & (n-1)^2 & \frac{(n-1)(3n-4)}{2} \\ \frac{n}{2} & n-1 & a \\ \frac{n(n-1)}{2} & (n-1)^2 & \frac{(n-1)(3n-4)}{2} \end{vmatrix}$$
$= 0$
($\because$ R_1 and R_3 are identical)

Hence, value of $\sum_{r=1}^{n-1} \Delta_r$ is independent of both 'a' and 'n'.

38. (c) Let $\Delta = \begin{vmatrix} a^2 & b^2 & c^2 \\ (a+\lambda)^2 & (b+\lambda)^2 & (c+\lambda)^2 \\ (a-\lambda)^2 & (b-\lambda)^2 & (c-\lambda)^2 \end{vmatrix}$

Apply $R_2 \to R_2 - R_3$

$$\Delta = \begin{vmatrix} a^2 & b^2 & c^2 \\ (a+\lambda)^2-(a-\lambda)^2 & (b+\lambda)^2-(b-\lambda)^2 & (c+\lambda)^2-(c-\lambda)^2 \\ (a-\lambda)^2 & (b-\lambda)^2 & (c-\lambda)^2 \end{vmatrix}$$

$$= \begin{vmatrix} a^2 & b^2 & c^2 \\ 4a\lambda & 4b\lambda & 4c\lambda \\ (a-\lambda)^2 & (b-\lambda)^2 & (c-\lambda)^2 \end{vmatrix}$$

$(\because (x+y)^2 - (x-y)^2 = 4xy)$

Taking out 4 common from R_2

$$= 4\begin{vmatrix} a^2 & b^2 & c^2 \\ a\lambda & b\lambda & c\lambda \\ a^2+\lambda^2-2a\lambda & b^2+\lambda^2-2b\lambda & c^2+\lambda^2-2c\lambda \end{vmatrix}$$

Apply $R_3 \to [R_3 - (R_1 - 2R_2)]$

$$= 4\begin{vmatrix} a^2 & b^2 & c^2 \\ a\lambda & b\lambda & c\lambda \\ \lambda^2 & \lambda^2 & \lambda^2 \end{vmatrix}$$

Taking out λ common from R_2 and λ^2 from R_3.

$$= 4\lambda(\lambda^2)\begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix} = k\lambda\begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$$

$\Rightarrow k = 4\lambda^2$

39. (b) $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ b & c & a \\ c & a & b \end{vmatrix}$

$$= (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ b & c & a \\ c & a & b \end{vmatrix}$$

$$= (a+b+c)\begin{vmatrix} 0 & 0 & 1 \\ b-c & c-a & a \\ c-a & a-b & b \end{vmatrix}$$

$= (a+b+c)[ab + bc + ca - a^2 - b^2 - c^2]$
$= -(a+b+c)[(a-b)^2 + (b-c)^2 + (c-a)^2]$

Since a, b, c are sides of a scalene triangle, therefore at least two of the a, b, c will be unequal.

$\therefore \quad (a-b)^2 + (b-c)^2 + (c-a)^2 > 0$

Also $a + b + c > 0$

$\therefore \quad -(a+b+c)[(a-b)^2 + (b-c)^2 + (c-a)^2] < 0$

40. **(c)** Let $\Delta = \begin{vmatrix} b^2+c^2 & ab & ac \\ ab & c^2+a^2 & bc \\ ac & bc & a^2+b^2 \end{vmatrix}$

Multiply C_1 by a, C_2 by b and C_3 by c and hence divide by abc.

$$= \frac{1}{abc}\begin{vmatrix} a(b^2+c^2) & ab^2 & ac^2 \\ a^2b & b(c^2+a^2) & bc^2 \\ a^2c & b^2c & c(a^2+b^2) \end{vmatrix}$$

Take out a, b, c common from R_1, R_2 and R_3 respectively.

$$\therefore \quad \Delta = \frac{abc}{abc}\begin{vmatrix} b^2+c^2 & b^2 & c^2 \\ a^2 & c^2+a^2 & c^2 \\ a^2 & b^2 & a^2+b^2 \end{vmatrix}$$

Apply $C_1 \to C_1 - C_2 - C_3$

$$\Delta = \begin{vmatrix} 0 & b^2 & c^2 \\ -2c^2 & c^2+a^2 & c^2 \\ -2b^2 & b^2 & a^2+b^2 \end{vmatrix}$$

$$= -2\begin{vmatrix} 0 & b^2 & c^2 \\ c^2 & c^2+a^2 & c^2 \\ b^2 & b^2 & a^2+b^2 \end{vmatrix}$$

Apply $C_2 - C_1$ and $C_3 - C_1$

$$= -2\begin{vmatrix} 0 & b^2 & c^2 \\ c^2 & a^2 & 0 \\ b^2 & 0 & a^2 \end{vmatrix} = -2\,[-b^2(c^2a^2) + c^2(-a^2b^2)]$$

$= 2a^2b^2c^2 + 2a^2b^2c^2 = 4a^2b^2c^2$

But $\Delta = ka^2b^2c^2 \;\therefore\; k = 4$

41. **(c)** Let $\Delta = \begin{vmatrix} -2a & a+b & a+c \\ b+a & -2b & b+c \\ c+a & b+c & -2c \end{vmatrix}$

Applying $C_1 + C_3$ and $C_2 + C_3$

$$\Delta = \begin{vmatrix} -a+c & 2a+b+c & a+c \\ 2b+a+c & -b+c & b+c \\ a-c & b-c & -2c \end{vmatrix}$$

Now, applying $R_1 + R_3$ and $R_2 + R_3$

$$\Delta = \begin{vmatrix} 0 & 2(a+b) & a-c \\ 2(a+b) & 0 & b-c \\ a-c & b-c & -2c \end{vmatrix}$$

On expanding, we get

$\Delta = -2(a+b)\,\{-2c\,[2(a+b)] - (a-c)(b-c)\} + (a-c)\,[2(a+b)(b-c)]$

$\Delta = 8c(a+b)(a+b) + 4(a+b)(a-c)(b-c)$

$= 4(a+b)\,[2ac + 2bc + ab - bc - ac + c^2]$

$= 4(a+b)\,[ac + bc + ab + c^2]$

$= 4(a+b)\,[c(a+c) + b(a+c)]$

$= 4(a+b)(b+c)(c+a)$

$= \alpha(a+b)(b+c)(c+a)$

Hence, $\alpha = 4$

42. **(b)** Vertices of triangle in complex form is

$z, iz, z + iz$

In cartesian form vertices are

$(x, y), (-y, x)$ and $(x-y, x+y)$

$$\therefore \text{ Area of triangle} = \frac{1}{2}\begin{vmatrix} x & y & 1 \\ -y & x & 1 \\ x-y & x+y & 1 \end{vmatrix}$$

$= \frac{1}{2}[x(x-x-y) - y(-y-x+y) + 1(-yx-y^2-x^2+xy)]$

$= \frac{1}{2}[-xy + xy - y^2 - x^2] = \frac{1}{2}(x^2+y^2)$

($\because$ Area can not be negative)

$= \frac{1}{2}|z|^2 \qquad (\because z = x+iy, |z|^2 = x^2+y^2)$

43. **(b)** Given parabola is $x^2 = 8y$

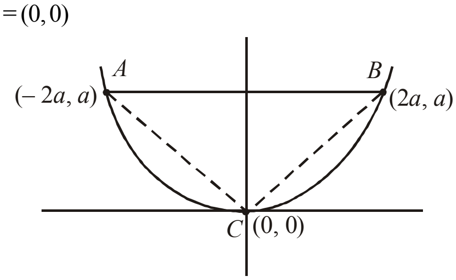
$\Rightarrow 4a = 8 \Rightarrow a = 2$

To find: Area of ΔABC

$A = (-2a, a) = (-4, 2)$

$B = (2a, a) = (4, 2)$

$C = (0, 0)$



$$\therefore \text{ Area} = \frac{1}{2}\begin{vmatrix} -4 & 2 & 1 \\ 4 & 2 & 1 \\ 0 & 0 & 1 \end{vmatrix} = \frac{1}{2}[-4(2) - 2(4) + 1(0)]$$

$= \frac{-16}{2} = -8 \approx 8$ sq. unit ($\because$ area cannot be negative)

44. **(b)**

$$\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^n c \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & a-1 & (-1)^{n+2}a \\ b+1 & b-1 & (-1)^{n+1}b \\ c-1 & c+1 & (-1)^n c \end{vmatrix} = 0$$

(Taking transpose of second determinant)

$$C_1 \Leftrightarrow C_3$$

$$\Rightarrow \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} - \begin{vmatrix} (-1)^{n+2}a & a-1 & a+1 \\ (-1)^{n+2}(-b) & b-1 & b+1 \\ (-1)^{n+2}c & c+1 & c-1 \end{vmatrix} = 0$$

$$C_2 \Leftrightarrow C_3$$

$$\Rightarrow \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + (-1)^{n+2}\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} = 0$$

$$\Rightarrow \left[1+(-1)^{n+2}\right]\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} = 0$$

$$C_2 - C_1,\ C_3 - C_1$$

$$\Rightarrow \left[1+(-1)^{n+2}\right]\begin{vmatrix} a & 1 & -1 \\ -b & 2b+1 & 2b-1 \\ c & -1 & 1 \end{vmatrix} = 0 \quad R_1 + R_3$$

$$\Rightarrow \left[1+(-1)^{n+2}\right]\begin{vmatrix} a+c & 0 & 0 \\ -b & 2b+1 & 2b-1 \\ c & -1 & 1 \end{vmatrix} = 0$$

$\Rightarrow$ $[1+(-1)^{n+2}](a+c)(2b+1+2b-1)=0$

$\Rightarrow$ $4b(a+c)[1+(-1)^{n+2}]=0$

$\Rightarrow$ $1+(-1)^{n+2}=0$ as $b(a+c) \neq 0$

$\Rightarrow$ n should be an odd integer.

45. **(d)** Given that, $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$\therefore D = \begin{vmatrix} 1 & 1 & 1 \\ 0 & x & 0 \\ 0 & 0 & y \end{vmatrix} = xy$$

Hence, D is divisible by both x and y

46. **(b)** Let r be the common ratio of an G.P., then

$$\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$$

$$= \begin{vmatrix} \log a_1 r^{n-1} & \log a_1 r^n & \log a_1 r^{n+1} \\ \log a_1 r^{n+2} & \log a_1 r^{n+3} & \log a_1 r^{n+4} \\ \log a_1 r^{n+5} & \log a_1 r^{n+6} & \log a_1 r^{n+7} \end{vmatrix}$$

$$= \begin{vmatrix} \log a_1 + (n-1)\log r & \log a_1 + n\log r & \log a_1 + (n+1)\log r \\ \log a_1 + (n+2)\log r & \log a_1 + (n+3)\log r & \log a_1 + (n+4)\log r \\ \log a_1 + (n+5)\log r & \log a_1 + (n+6)\log r & \log a_2 + (n+7)\log r \end{vmatrix}$$

Applying $C_3 \to C_3 + C_1$, we get

$$= \begin{vmatrix} \log a_1 + (n-1)\log r & \log a_1 + n\log r & 2[\log a_1 + n\log r] \\ \log a_1 + (n+2)\log r & \log a_1 + (n+3)\log r & 2[\log a_1 + (n+3)\log r] \\ \log a_1 + (n+5)\log r & \log a_1 + (n+6)\log r & 2[\log a_1 + (n+6)\log r] \end{vmatrix}$$

$= 0$

47. **(d)** Applying, $C_1 \to C_1 + C_2 + C_3$, we get

$$f(x) = \begin{vmatrix} 1+(a^2+b^2+c^2+2)x & (1+b^2)x & (1+c^2)x \\ 1+(a^2+b^2+c^2+2)x & 1+b^2x & (1+c^2)x \\ 1+(a^2+b^2+c^2+2)x & (1+b^2)x & 1+c^2x \end{vmatrix}$$

$[\because a^2+b^2+c^2=-2]$

$$= \begin{vmatrix} 1 & (1+b^2)x & (1+c^2)x \\ 1 & 1+b^2x & (1+c^2)x \\ 1 & (1+b^2)x & 1+c^2x \end{vmatrix}$$

Applying, $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$\therefore f(x) = \begin{vmatrix} 1 & (1+b^2)x & (1+c^2)x \\ 0 & 1-x & 0 \\ 0 & 0 & 1-x \end{vmatrix}$$

$f(x) = (x-1)^2$

Hence degree $= 2$.

48. **(d)** Let r be the common ratio of an G.P., then

$$\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$$

$$= \begin{vmatrix} \log a_1 r^{n-1} & \log a_1 r^n & \log a_1 r^{n+1} \\ \log a_1 r^{n+2} & \log a_1 r^{n+3} & \log a_1 r^{n+4} \\ \log a_1 r^{n+5} & \log a_1 r^{n+6} & \log a_1 r^{n+7} \end{vmatrix}$$

$$= \begin{vmatrix} \log a_1 + (n-1)\log r & \log a_1 + n\log r & \log a_1 + (n+1)\log r \\ \log a_1 + (n+2)\log r & \log a_1 + (n+3)\log r & \log a_1 + (n+4)\log r \\ \log a_1 + (n+5)\log r & \log a_1 + (n+6)\log r & \log a_2 + (n+7)\log r \end{vmatrix}$$

Applying $C_3 \to C_3 + C_1$, we get

$$= \begin{vmatrix} \log a_1 + (n-1)\log r & \log a_1 + n\log r & 2[\log a_1 + n\log r] \\ \log a_1 + (n+2)\log r & \log a_1 + (n+3)\log r & 2[\log a_1 + (n+3)\log r] \\ \log a_1 + (n+5)\log r & \log a_1 + (n+6)\log r & 2[\log a_1 + (n+6)\log r] \end{vmatrix}$$

$= 0$

49. (c) Given that $\begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ ax+b & bx+c & 0 \end{vmatrix}$

Applying $R_3 \to R_3 - (xR_1 + R_2)$;

$$= \begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ 0 & 0 & -(ax^2+2bx+c) \end{vmatrix}$$

$= (ax^2+2bx+c)(b^2-ac) = (+)(-) = -\text{ve}$.

[Given that discriminant of $ax^2+2bx+c$ is $-$ve

$\therefore 4b^2 - 4ac < 0 \Rightarrow b^2 - ac < 0$]

50. (d) $l = AR^{p-1} \Rightarrow \log l = \log A + (p-1)\log R$

$m = AR^{q-1} \Rightarrow \log m = \log A + (q-1)\log R$

$n = AR^{r-1} \Rightarrow \log n = \log A + (r-1)\log R$

Now, $\begin{vmatrix} \log l & p & 1 \\ \log m & q & 1 \\ \log n & r & 1 \end{vmatrix} = \begin{vmatrix} \log A + (p-1)\log R & p & 1 \\ \log A + (q-1)\log R & q & 1 \\ \log A + (r-1)\log R & r & 1 \end{vmatrix}$

Operating

$C_1 - (\log R)C_2 + (\log R - \log A)C_3 = \begin{vmatrix} 0 & p & 1 \\ 0 & q & 1 \\ 0 & r & 1 \end{vmatrix} = 0$

51. (a) $|\text{adj } A| = |A|^2 = 9$

$[\because |\text{adj } A| = |A|^{n-1}]$

$\Rightarrow |A| = \pm 3 = \lambda \Rightarrow |\lambda| = 3$

$\Rightarrow |B| = |\text{adj } A|^2 = 81$

$\mu = |(B^{-1})^T| = |B^{-1}| = |B|^{-1} = \dfrac{1}{|B|} = \dfrac{1}{81}$

52. (a) $|A| = \begin{vmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{vmatrix} = ((9+4) - 1(3-4) + 2(-1-3))$

$= 13 + 1 - 8 = 6$

$|adjB| = |adj(adjA)| = |A|^{(n-1)^2} = |A|^4 = (36)^2$

$|C| = |3A| = 3^3 \times 6$

Hence, $\dfrac{|adjB|}{|C|} = \dfrac{36 \times 36}{3^3 \times 6} = 8$

53. (c) $\because B = A^{-1} = \Rightarrow |B| = \dfrac{1}{|A|}$

Now, $|B| = \begin{vmatrix} 5 & 2\alpha & 1 \\ 0 & 2 & 1 \\ \alpha & 3 & -1 \end{vmatrix} = 2\alpha^2 - 2\alpha - 25$

Given, det. $(A) + 1 = 0$

$\Rightarrow \dfrac{1}{2\alpha^2 - 2\alpha - 25} + 1 = 0$

$\Rightarrow \dfrac{2\alpha^2 - 2\alpha - 24}{2\alpha^2 - 2\alpha - 25} = 0$

$\Rightarrow \alpha = 4, -3 \Rightarrow$ Sum of values $= 1$

54. (b)

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}\cdots\begin{bmatrix} 1 & n-1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 78 \\ 0 & 1 \end{bmatrix}$$

$\Rightarrow \begin{bmatrix} 1 & 1+2+3+\ldots+(n-1) \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 78 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \dfrac{(n-1)n}{2} = 78 \Rightarrow n^2 - n - 15 = 0$

$\Rightarrow n = 13$

Now, the matrix $\begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 13 \\ 0 & 1 \end{bmatrix}$

Then, the required inverse of $\begin{bmatrix} 1 & 13 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -13 \\ 0 & 1 \end{bmatrix}$

55. (c) Let $|A| = a, |B| = b$

$\Rightarrow |A^T| = a\, |A^{-1}| = \dfrac{1}{a}, |B^T| = b, |B^{-1}| = \dfrac{1}{b}$

$\because |ABA^T| = 8 \Rightarrow |A|\,|B|\,|A^T| = 8 \ldots(1)$

$\Rightarrow a.b.a = 8 \Rightarrow a^2 b = 8$

$\because |AB^{-1}| = 8 \Rightarrow |A|\,|B^{-1}| = 8 \Rightarrow a \,.\, \dfrac{1}{b} = 8$ ...(2)

From (1) & (2)

$a = 4, b = \dfrac{1}{2}$

Then, $|BA^{-1}B^T| = |B|\,|A^{-1}|\,|B^T| = b \,.\, \dfrac{1}{a} \,.\, b = \dfrac{b^2}{a} = \dfrac{1}{16}$

56. (c) $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \Rightarrow |A| = 1$

$\text{adj}(A) = \begin{bmatrix} +\cos\theta & -\sin\theta \\ +\sin\theta & +\cos\theta \end{bmatrix}^T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$\Rightarrow A^{-1} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = B$

$$B^2=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$$

$$=\begin{bmatrix}\cos 2\theta & \sin 2\theta\\ -\sin 2\theta & \cos 2\theta\end{bmatrix}$$

$$\Rightarrow \quad B^3=\begin{bmatrix}\cos 3\theta & \sin 3\theta\\ -\sin 3\theta & \cos 3\theta\end{bmatrix}$$

$$\Rightarrow \quad A^{-50}=B^{50}=\begin{bmatrix}\cos(50\theta) & \sin(50\theta)\\ -\sin(50\theta) & \cos(50\theta)\end{bmatrix}$$

$$(A^{-50})_{\theta=\frac{\pi}{12}}=\begin{bmatrix}\frac{\sqrt{3}}{2} & \frac{1}{2}\\ -\frac{1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}$$

$$\left[\because \cos\left(\frac{50\pi}{12}\right)=\cos\left(4\pi+\frac{\pi}{6}\right)=\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}\right]$$

57. **(d)** Since $A.\begin{bmatrix}1 & 2\\ 0 & 3\end{bmatrix}$ is a scalar matrix and $|3A|=108$

suppose the scalar matrix is $\begin{bmatrix}k & 0\\ 0 & k\end{bmatrix}$

$$\therefore A.\begin{bmatrix}1 & 2\\ 0 & 3\end{bmatrix}=\begin{bmatrix}k & 0\\ 0 & k\end{bmatrix}$$

$$\Rightarrow A=\begin{bmatrix}k & 0\\ 0 & k\end{bmatrix}\begin{bmatrix}1 & 2\\ 0 & 3\end{bmatrix}^{-1}$$

$[\therefore\ AB=C\Rightarrow ABB^{-1}=CB^{-1}\Rightarrow A=CB^{-1}]$

$$\Rightarrow A=\frac{1}{3}\begin{bmatrix}k & 0\\ 0 & k\end{bmatrix}\begin{bmatrix}3 & -2\\ 0 & 1\end{bmatrix}$$

$$\Rightarrow A=\begin{bmatrix}k & 0\\ 0 & k\end{bmatrix}\begin{bmatrix}1 & -\frac{2}{3}\\ 0 & \frac{1}{3}\end{bmatrix}$$

$$\Rightarrow A=\begin{bmatrix}k & -\frac{2}{3}k\\ 0 & \frac{k}{3}\end{bmatrix} \quad ...(1)$$

$\because\ |3A|=108$

$$\Rightarrow 108=\begin{vmatrix}3k & -2k\\ 0 & k\end{vmatrix}$$

$\Rightarrow 3k^2=108 \Rightarrow k^2=36 \quad \Rightarrow k=\pm 6$

For $k=6$

$$A=\begin{bmatrix}6 & -4\\ 0 & 2\end{bmatrix} \text{ ... From (1)}$$

$$\Rightarrow A^2=\begin{bmatrix}36 & -32\\ 0 & 4\end{bmatrix}$$

For $k=-6$

$$\Rightarrow A=\begin{bmatrix}-6 & 4\\ 0 & -2\end{bmatrix} \text{ .... From (1)}$$

$$\Rightarrow A^2=\begin{bmatrix}36 & -32\\ 0 & 4\end{bmatrix}$$

58. **(a)** We have
$(A-3I)(A-5I)=O$
$\Rightarrow \quad A^2-8A+15I=O$
Multiplying both sides by A^{-1}, we get;
$A^{-1}A.A-8A^{-1}A+15A^{-1}I=A^{-1}O$
$\Rightarrow \quad A-8I+15A^{-1}=O$
$A+15A^{-1}=8I$

$$\frac{A}{2}+\frac{15A^{-1}}{2}=4I$$

$$\therefore \quad \alpha+\beta=\frac{1}{2}+\frac{15}{2}=\frac{16}{2}=8$$

59. **(c)** We have $A=\begin{bmatrix}2 & -3\\ -4 & 1\end{bmatrix}$

$$\Rightarrow A^2=\begin{bmatrix}16 & -9\\ -12 & 13\end{bmatrix} \Rightarrow 3A^2=\begin{bmatrix}48 & -27\\ -36 & 39\end{bmatrix}$$

Also $12A=\begin{bmatrix}24 & -36\\ -48 & 12\end{bmatrix}$

$$\therefore 3A^2+12A=\begin{bmatrix}48 & -27\\ -36 & 39\end{bmatrix}+\begin{bmatrix}24 & -36\\ -48 & 12\end{bmatrix}$$

$$=\begin{bmatrix}72 & -63\\ -84 & 51\end{bmatrix}$$

$$\text{adj}(3A^2+12A)=\begin{bmatrix}51 & 63\\ 84 & 72\end{bmatrix}$$

60. **(b)**

61. **(d)** Given that $A(\text{adj } A) = A A^T$

Pre-multiply by A^{-1} both side, we get

$\Rightarrow A^{-1}A(\text{adj } A) = A^{-1}A A^T$

$\text{adj } A = A^T$

$$\Rightarrow \begin{bmatrix}2 & b\\ -3 & 5a\end{bmatrix}=\begin{bmatrix}5a & 3\\ -b & 2\end{bmatrix}$$

$\Rightarrow a=\frac{2}{5}$ and $b=3$

$\Rightarrow 5a+b=5$

62. (a) $A^2-5A=-7I$

$AAA^{-1}-5AA^{-1}=-7IA^{-1}$

$AI-5I=-7A^{-1}$

$A-5I=-7A^{-1}$

$A^{-1}=\frac{1}{7}(5I-A)$

$A^3-2A^2-3A+I=A(5A-7I)-2A^2-3A+I$

$=5A^2-7A-2A^2-3A+I=3A^2-10A+I$

$=3(5A-7I)-10A+I=5A-20I=5(A-4I)$

63. (a) $|5.\text{ adj A}|=5\Rightarrow 5^3.|A|^{3-1}=5$

$\Rightarrow 125|A|^2=5\Rightarrow |A|=\pm\frac{1}{5}$

64. (d) $BB'=B(A^{-1}A')'=B(A')'(A^{-1})'$

$=BA(A^{-1})'=(A^{-1}A')(A(A^{-1})')$

$=A^{-1}A.A'.(A^{-1})'$ $\{\text{as } AA'=A'A\}$

$=I(A^{-1}A)'=I\cdot I=I^2=I$

65. (a) Given $A\begin{bmatrix}1&2&3\\0&2&3\\0&1&1\end{bmatrix}=\begin{bmatrix}0&0&1\\1&0&0\\0&1&0\end{bmatrix}$

Applying $C_1\leftrightarrow C_3$

$A\begin{bmatrix}3&2&1\\3&2&0\\1&1&0\end{bmatrix}=\begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix}$

Again Applying $C_2\leftrightarrow C_3$

$A\begin{bmatrix}3&1&2\\3&0&2\\1&0&1\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$

pre-multiplying both sides by A^{-1}

$A^{-1}A\begin{bmatrix}3&1&2\\3&0&2\\1&0&1\end{bmatrix}=A^{-1}\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$

$I\begin{bmatrix}3&1&2\\3&0&2\\1&0&1\end{bmatrix}=A^{-1}I=A^{-1}$

$(\because A^{-1}A=I \text{ and } I=\text{Identity matrix})$

Hence, $A^{-1}=\begin{bmatrix}3&1&2\\3&0&2\\1&0&1\end{bmatrix}$

66. (b) $|P|=1(12-12)-\alpha(4-6)+3(4-6)=2\alpha-6$

Now, adj A = P $\Rightarrow$ |adj A| = |P|

$\Rightarrow |A|^2=|P|$

$\Rightarrow |P|=16$

$\Rightarrow 2\alpha-6=16$

$\Rightarrow \alpha=11$

67. (c) Given that $P^3=Q^3$ ...(1)

and $P^2Q=Q^2P$ ...(2)

Subtracting (1) and (2), we get

$P^3-P^2Q=Q^3-Q^2P$

$\Rightarrow P^2(P-Q)+Q^2(P-Q)=0$

$\Rightarrow (P^2+Q^2)(P-Q)=0$

$\because P\neq Q,\ \therefore P^2+Q^2=0$

Hence $|P^2+Q^2|=0$

68. (d) Let $Au_1=\begin{pmatrix}1\\0\\0\end{pmatrix}$ and $Au_2=\begin{pmatrix}0\\1\\0\end{pmatrix}$

Then, $Au_1+Au_2=\begin{pmatrix}1\\0\\0\end{pmatrix}+\begin{pmatrix}0\\1\\0\end{pmatrix}$

$\Rightarrow A(u_1+u_2)=\begin{pmatrix}1\\1\\0\end{pmatrix}$ ...(1)

Given that $A=\begin{pmatrix}1&0&0\\2&1&0\\3&2&1\end{pmatrix}$

$\Rightarrow |A|=1(1)-0(2)+0(4-3)=1$

$C_{11}=1$ $C_{21}=0$ $C_{31}=0$

$C_{12}=-2$ $C_{22}=1$ $C_{32}=0$

$C_{13}=1$ $C_{23}=-2$ $C_{33}=1$

$\therefore adjA=\begin{bmatrix}1&0&0\\-2&0&0\\1&-2&1\end{bmatrix}$

We know,

$A^{-1}=\frac{1}{|A|}adjA$

$\Rightarrow A^{-1}=adj(A)\quad (\because |A|=1)$

Now, from equation (1), we have

$u_1+u_2=A^{-1}\begin{pmatrix}1\\1\\0\end{pmatrix}$

$=\begin{bmatrix}1&0&0\\-2&1&0\\1&-2&1\end{bmatrix}\begin{pmatrix}1\\1\\0\end{pmatrix}=\begin{bmatrix}1\\-1\\-1\end{bmatrix}$

69. (c) $A=\begin{bmatrix}0&0&a\\0&b&c\\d&e&f\end{bmatrix}$, $|A|=-abd\neq 0$

$c_{11} = +(bf - ce)$, $c_{12} = -(-cd) = cd$, $c_{13} = +(-bd) = -bd$
$c_{21} = -(-ea) = ae$, $c_{22} = +(-ad) = -ad$, $c_{23} = -(0) = 0$
$c_{31} = +(-ab) = -ab$, $c_{32} = -(0) = 0$, $c_{33} = 0$

$$\text{Adj } A = \begin{bmatrix} (bf-ce) & ae & -ab \\ cd & -ad & 0 \\ -bd & 0 & 0 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{abd}\begin{bmatrix} bf-ce & ae & -ab \\ cd & -ad & 0 \\ -bd & 0 & 0 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 0 & 0 & d \\ 0 & b & e \\ a & c & f \end{bmatrix}$$

Now $A^{-1} = A^T$

$$\Rightarrow \frac{1}{-abd}\begin{bmatrix} bf-ce & ae & -ab \\ cd & -ad & 0 \\ -bd & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & d \\ 0 & b & e \\ a & c & f \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} bf-ce & ae & -ab \\ cd & -ad & 0 \\ -bd & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -abd^2 \\ 0 & -ab^2d & -abde \\ -a^2bd & -abcd & -abdf \end{bmatrix}$$

$\therefore\ bf - ce = ae = cd = 0$ ...(i)
$abd^2 = ab,\ ab^2d = ad,\ a^2bd = bd$ ...(ii)
$abde = abcd = abdf = 0$ ...(iii)
From (ii),
$(abd^2).(ab^2d).(a^2bd) = ab.\ ad.\ bd$
$\Rightarrow (abd)^4 - (abd)^2 = 0$
$\Rightarrow (abd)^2\ [(abd)^2 - 1] = 0$
$\because abd \neq 0,\ \therefore\ abd = \pm 1$ ...(iv)
From (iii) and (iv),
$e = c = f = 0$ ...(v)
From (i) and (v),
$bf = ae = cd = 0$ ...(vi)
From (iv), (v) and (vi), it is clear that a, b, d can be any non-zero integer such that $abd = \pm 1$
But it is only possible, if $a = b = d = \pm 1$
Hence, there are 2 choices for each a, b and d. there fore, there are $2\times2\times2$ choices for a, b and d. Hence number of required matrices $= 2\times2\times2 = (2)^3$

70. **(a)** Let A and B be real matrices such that $A = \begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix}$

and $B = \begin{bmatrix} 0 & \gamma \\ \delta & 0 \end{bmatrix}$

Now, $AB = \begin{bmatrix} 0 & \alpha\gamma \\ \beta\delta & 0 \end{bmatrix}$

and $BA = \begin{bmatrix} 0 & \gamma\beta \\ \delta\alpha & 0 \end{bmatrix}$

Statement - 1 :

$$AB - BA = \begin{bmatrix} 0 & \gamma(\alpha-\beta) \\ \delta(\beta-\alpha) & 0 \end{bmatrix}$$

$|AB - BA| = (\alpha-\beta)^2\gamma\delta \neq 0$
$\therefore\ AB - BA$ is always an invertible matrix.
Hence, statement - 1 is true.
But $AB - BA$ can be identity matrix if $\gamma = -\delta$ or $\delta = -\gamma$
So, statement - 2 is false.

71. **(b)** **For reflexive**

$A = P^{-1}AP$ is true,
For $P = I$, which is an invertible matrix.
$(A, A) \in R$
$\therefore$ R is reflexive.

For symmetry

As $(A, B) \in R$ for matrix P
$A = P^{-1}BP$
$\Rightarrow PAP^{-1} = B$
$\Rightarrow B = PAP^{-1}$
$\Rightarrow B = (P^{-1})^{-1}A(P^{-1})$
$\therefore$ $(B, A) \in R$ for matrix P^{-1}
$\therefore$ R is symmetric.

For transitivity

$A = P^{-1}BP$
and $B = P^{-1}CP$
$\Rightarrow A = P^{-1}(P^{-1}CP)P$
$\Rightarrow A = (P^{-1})^2 CP^2$
$\Rightarrow A = (P^2)^{-1}C(P^2)$
$\therefore$ $(A, C) \in R$ for matrix P^2
$\therefore$ R is transitive.
So R is equivalence.
So, statement-1 is true.
We know that if A and B are two invertible matrices of order n, then
$(AB)^{-1} = B^{-1}A^{-1}$
So, statement-2 is true.

72. **(a)** We know that if A is square matrix of order n then
$adj\,(adj\,A) = |A|^{n-2}A$.
$= |A|^0 A = A$

Also $|adj\,A| = |A|^{n-1} = |A|^{2-1} = |A|$

$\therefore$ Both the statements are true but statement-2 is not a correct explanation for statement-1 .

73. **(c)** Given that all entries of square matrix A are integers, therefore all cofactors should also be integers.
If det $A = \pm 1$ then A^{-1} exists. Also all entries of A^{-1} are integers.

74. **(d)** Given that $A^2 - A + I = 0$
Pre-multiply by A^{-1} both side, we get
$A^{-1}A^2 - A^{-1}A + A^{-1}.I = A^{-1}.0$
$\Rightarrow A - I + A^{-1} = 0$ or $A^{-1} = I - A$.

75. **(a)** Given that $10B = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$

$$\Rightarrow B = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$$

Given that $B = A^{-1} \Rightarrow AB = I$

$$\Rightarrow \frac{1}{10}\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \frac{1}{10}\begin{bmatrix} 10 & 0 & 5-2 \\ 0 & 10 & -5+\alpha \\ 0 & 0 & 5+\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \frac{5-\alpha}{10} = 0 \Rightarrow \alpha = 5$$

76. **(a)** Given that $A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

clearly $A \neq 0$. Also $|A| = -1 \neq 0$

$$\therefore A^{-1} \text{ exists, further } (-1)I = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \neq A$$

$$\text{Also } A^2 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

77. **(c)** $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & \lambda \end{vmatrix} = 0 \Rightarrow \lambda = 5$

$$D_x = \begin{vmatrix} 2 & 1 & 1 \\ 5 & 2 & 3 \\ \mu & 3 & 5 \end{vmatrix} = 0 \Rightarrow \mu = 8$$

78. **(3.00)**
For non-zero solution, $\Delta = 0$

$$\Rightarrow \begin{vmatrix} \lambda-1 & 3\lambda+1 & 2\lambda \\ \lambda-1 & 4\lambda-2 & \lambda+3 \\ 2 & 3\lambda+1 & 3(\lambda-1) \end{vmatrix} = 0$$

$\Rightarrow 6\lambda^3 - 36\lambda^2 + 54\lambda = 0$

$\Rightarrow 6\lambda[\lambda^2 - 6\lambda + 9] = 0$

$\Rightarrow \lambda = 0, \lambda = 3$ [Distinct values]

Then, the sum of distinct values of $\lambda = 0 + 3 = 3$.

79. **(a)** $\because \begin{vmatrix} 2 & -4 & \lambda \\ 1 & -6 & 1 \\ \lambda & -10 & 4 \end{vmatrix} = 0 \Rightarrow 3\lambda^2 - 7\lambda - 12 = 0$

$\Rightarrow \lambda = 3$ or $-\frac{2}{3}$

$$D_1 = \begin{vmatrix} 1 & -4 & \lambda \\ 2 & -6 & 1 \\ 3 & -10 & 4 \end{vmatrix} = 2(3-\lambda)$$

$\therefore$ When $\lambda = -\frac{2}{3}$, $D_1 \neq 0$.

Hence, equations will be inconsistent when $\lambda = -\frac{2}{3}$.

80. **(a)** Since, system of linear equations has non-zero solution

$\therefore \Delta = 0$

$$\Rightarrow \begin{vmatrix} 1 & 1 & 3 \\ 1 & 3 & k^2 \\ 3 & 1 & 3 \end{vmatrix} = 0$$

$\Rightarrow 1(9-k^2) - 1(3-3k^2) + 3(1-9) = 0$

$\Rightarrow 9 - k^2 - 3 + 3k^2 - 24 = 0$

$\Rightarrow 2k^2 = 18 \Rightarrow k^2 = 9, k = \pm 3$

So, equations are

$x + y + 3z = 0$ ...(i)

$x + 3y + 9z = 0$ ...(ii)

$3x + y + 3z = 0$ ...(iii)

Now, from equation (i) – (ii),

$-2y - 6z = 0 \Rightarrow y = -3z \Rightarrow \frac{y}{z} = -3$ ...(iv)

Now, from equation (i) – (iii),

$-2x = 0 \Rightarrow x = 0$

So, $x + \frac{y}{z} = 0 - 3 = -3$

81. **(5.00)**
For infinitely many solutions,

$\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

$$\Delta = \begin{vmatrix} 1 & -2 & 3 \\ 2 & 1 & 1 \\ 1 & -7 & a \end{vmatrix} = 0$$

$\Rightarrow (a+7) - 2(1-2a) + 3(-15) = 0$

$\Rightarrow a = 8$

$$\Delta_3 = \begin{vmatrix} 1 & -2 & 9 \\ 2 & 1 & b \\ 1 & -7 & 24 \end{vmatrix} = 0$$

$\Rightarrow (24 + 7b) - 2(b - 48) + 9(-15) = 0$

$\Rightarrow b = 3$

$\therefore a - b = 5.$

82. **(b)** Given that $Ax = b$ has solutions x_1, x_2, x_3 and b is equal to b_1, b_2 and b_3

$\therefore x_1 + y_1 + z_1 = 1$

$\Rightarrow 2y_1 + z_1 = 2 \Rightarrow z_1 = 2$

Determinant of coefficient matrix

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{vmatrix} = 2$$

83. **(d)** $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & -1 \\ 3 & 2 & \lambda \end{vmatrix} = 0$ [$\because$ Equation has many solutions]

$\Rightarrow -15 + 6 + 2\lambda = 0 \Rightarrow \lambda = \frac{9}{2}$

$$\therefore D_Z = \begin{vmatrix} 1 & 1 & 2 \\ 2 & 4 & 6 \\ 3 & 2 & 2\mu \end{vmatrix} = 0 \Rightarrow \mu = 5$$

$\therefore 2\lambda + \mu = 14.$

84. **(8)**
The given system of equations

$x - 2y + 5z = 0$ ...(i)

$-2x + 4y + z = 0$ ...(ii)

$-7x + 14y + 9z = 0$ ...(iii)

From equation, $2 \times$ (i) + (ii) $\Rightarrow z = 0$

Put $z = 0$ in equation (i), we get $x = 2y$

$\because 15 \le x^2 + y^2 + z^2 \le 150$

$\Rightarrow 15 \le 4y^2 + y^2 \le 150$

$[\because x = 2y,\ z = 0]$

$\Rightarrow 3 \le y^2 \le 30$

$\Rightarrow y = \pm 2, \pm 3, \pm 4, \pm 5$

$\Rightarrow$ 8 solutions.

85. **(d)** $\Delta = \begin{vmatrix} 2 & -1 & 2 \\ 1 & -2 & \lambda \\ 1 & \lambda & 1 \end{vmatrix} = -(\lambda - 1)(2\lambda + 1)$

$$\Delta_1 = \begin{vmatrix} 2 & -1 & 2 \\ -4 & -2 & \lambda \\ 4 & \lambda & 1 \end{vmatrix} = -2(\lambda^2 + 6\lambda - 4)$$

For no solution $\Delta = 0$ and at least one of Δ_1, Δ_2 and Δ_3 is non-zero.

$\therefore \Delta = 0 \Rightarrow \lambda = 1, -\frac{1}{2}$ and $\Delta_1 \ne 0$

Hence, $S = \left\{1, -\frac{1}{2}\right\}$

86. **(d)** $\because |P| = 1(-3+36) - 2(2+4) + 1(-18-3) = 0$

Given that $PX = 0$

$\therefore$ System of equations

$x + 2y + z = 0$; $2x - 3y + 4z = 0$

and $x + 9y - z = 0$ has infinitely many solution.

Let $z = k \in \mathbf{R}$ and solve above equations, we get

$x = -\frac{11k}{7},\ y = \frac{2k}{7},\ z = k$

But given that $x^2 + y^2 + z^2 = 1$

$\therefore k = \pm \frac{7}{\sqrt{174}}$

$\therefore$ Two solutions only.

87. **(c)** The given system of linear equations

$7x + 6y - 2z = 0$ ...(i)

$3x + 4y + 2z = 0$ ...(ii)

$x - 2y - 6z = 0$ ...(iii)

Now, determinant of coefficient matrix

$$\Delta = \begin{vmatrix} 7 & 6 & -2 \\ 3 & 4 & 2 \\ 1 & -2 & -6 \end{vmatrix}$$

$= 7(-20) - 6(-20) - 2(-10)$

$= -140 + 120 + 20 = 0$

So, there are infinite non-trivial solutions.

From eqn. (i) + 3 × (iii); we get

$10x - 20z = 0 \Rightarrow x = 2z$

Hence, there are infinitely many solutions (x, y, z) satisfying $x = 2z$.

88. (a) From the given linear equation, we get

$$D = \begin{vmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 4 & 4 \end{vmatrix} (R_3 \to R_3 - 2R_2 + 3R_3)$$

$$= \begin{vmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 0 & 0 & 0 \end{vmatrix} = 0$$

Now, let $P_3 = 4x + 4y + 4z - \delta = 0$. If the system has solutions it will have infinite solution.

So, $P_3 \equiv \alpha P_1 + \beta P_2$

Hence, $3\alpha + \beta = 4$ and $4\alpha + 2\beta = 4$

$\Rightarrow \alpha = 2$ and $\beta = -2$

So, for infinite solution $2\mu - 2 = \delta$

$\Rightarrow$ For $2\mu \neq \delta + 2$ system is inconsistent

89. (c) $D = \begin{vmatrix} \lambda & 2 & 2 \\ 2\lambda & 3 & 5 \\ 4 & \lambda & 6 \end{vmatrix}$

$D = \lambda^2 + 6\lambda - 16$

$D = (\lambda + 8)(2 - \lambda)$

For no solutions, D = 0

$\Rightarrow \lambda = -8, 2$

when $\lambda = 2$

$$D_1 = \begin{vmatrix} 5 & 2 & 2 \\ 8 & 3 & 5 \\ 10 & 2 & 6 \end{vmatrix}$$

$= 5[18 - 10] - 2[48 - 50] + 2(16 - 30]$

$= 40 + 4 - 28 \neq 0$

There exist no solutions for $\lambda = 2$

90. (a) For non-zero solution

$$\begin{vmatrix} 2 & 2a & a \\ 2 & 3b & b \\ 2 & 4c & c \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0$$

$\Rightarrow (3bc - 4bc) - (2ac - 4ac) + (2ab - 3ab) = 0$

$\Rightarrow -bc + 2ac - ab = 0$

$\Rightarrow ab + bc + 2ac$

$\Rightarrow \frac{2}{b} = \frac{1}{a} + \frac{1}{c}$

$\Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ in A.P.

91. (13) $x + y + z = 6$ ...(i)

$x + 2y + 3z = 10$ ...(ii)

$3x + 2y + \lambda z = \mu$ ...(iii)

From (i) and (ii),

If $z = 0 \Rightarrow x + y = 6$ and $x + 2y = 10$

$\Rightarrow y = 4, x = 2$

$(2, 4, 0)$

If $y = 0 \Rightarrow x + z = 6$ and $x + 3z = 10$

$\Rightarrow z = 2$ and $x = 4$

$(4, 0, 2)$

So, $3x + 2y + \lambda z = \mu$, must pass through (2, 4, 0) and (4, 0, 2)

So, $6 + 8 = \mu \Rightarrow \mu = 14$

and $12 + 2\lambda = \mu$

$12 + 2\lambda = 14 \Rightarrow \lambda = 1$

So, $\mu - \lambda^2 = 14 - 1 = 13$

92. (d) Given system of linear equations: $x + y + z = 5$; $x + 2y + 2z = 6$ and $x + 3y + \lambda z = \mu$ have infinite solution.

$\therefore \Delta = 0, \Delta x = \Delta y = \Delta z = 0$

Now, $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 3 & \lambda \end{vmatrix} = 0$

$\Rightarrow 1(2\lambda - 6) - 1(\lambda - 2) + 1(3 - 2) = 0$

$\Rightarrow 2\lambda - 6 - \lambda + 2 + 1 = 0 \Rightarrow \lambda = 3$

$$\Delta y = \begin{vmatrix} 1 & 5 & 1 \\ 1 & 6 & 2 \\ 1 & \mu & 3 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 1 & 5 & 1 \\ 0 & 1 & 1 \\ 0 & \mu - 5 & 2 \end{vmatrix} = 0$$

$\Rightarrow 1(2 - \mu + 5) = 0 \Rightarrow \mu = 7$

$\therefore \lambda + \mu = 10$

93. (d) $\because$ system of equations has infinitely many solutions.

$\therefore \Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

Here, $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 4 & \lambda & -\lambda \\ 3 & 2 & -4 \end{vmatrix} = 0$

$C_1 \to C_1 - C_2$ & $C_2 \to C_2 - C_3$

$$\Rightarrow \begin{vmatrix} 0 & 0 & 1 \\ 4 - \lambda & 2\lambda & -\lambda \\ 1 & 6 & -4 \end{vmatrix} = 0 \Rightarrow \lambda = 3$$

Now, for $\lambda = 3$, $\Delta_1 = \begin{vmatrix} 6 & 1 & 1 \\ \lambda-2 & \lambda & -\lambda \\ -5 & 2 & -4 \end{vmatrix} = 0$

For $\lambda = 3$, $\Delta_2 = \begin{vmatrix} 1 & 6 & 1 \\ 4 & \lambda-2 & -\lambda \\ 3 & -5 & -4 \end{vmatrix} = 0$

For $\lambda = 3$, $\Delta_3 = \begin{vmatrix} 1 & 1 & 6 \\ 4 & \lambda & \lambda-2 \\ 3 & 2 & -5 \end{vmatrix} = 0$

$\therefore$ for $\lambda = 3$, system of equations has infinitely many solutions.

94. **(b)** Given system of equations has a non-trivial solution.

$$\Rightarrow \Delta = 0 \Rightarrow \begin{vmatrix} 2 & 3 & -1 \\ 1 & k & -2 \\ 2 & -1 & 1 \end{vmatrix} = 0 \Rightarrow k = \frac{9}{2}$$

$\therefore$ equations are $2x + 3y - z = 0$ ...(i)

$2x - y + z = 0$ ...(ii)

$2x + 9y - 4z = 0$ ...(iii)

By (i) – (ii), $2y = z$

$\therefore z = -4x$ and $2x + y = 0$

$$\therefore \frac{x}{y} + \frac{y}{z} + \frac{z}{x} + k = \frac{-1}{2} + \frac{1}{2} - 4 + \frac{9}{2} = \frac{1}{2}$$

95. **(b)** If the system of equations has non-trivial solutions, then the determinant of coefficient matrix is zero

$$\begin{vmatrix} 1 & -c & -c \\ c & -1 & c \\ c & c & -1 \end{vmatrix} = 0$$

$\Rightarrow (1 - c^2) + c(-c - c^2) - c(c^2 + c) = 0$

$\Rightarrow (1 + c)(1 - c) - 2c^2(1 + c) = 0$

$\Rightarrow (1 + c)(1 - c - 2c^2) = 0$

$\Rightarrow (1 + c)^2(1 - 2c) = 0$

$\Rightarrow c = -1$ or $\frac{1}{2}$

Hence, the greatest value of c is $\frac{1}{2}$ for which the system of linear equations has non-trivial solution.

96. **(b)** Given system of linear equations,

$x - 2y + kz = 1, 2x + y + z = 2, 3x - y - kz = 3$

$$\Delta = \begin{vmatrix} 1 & -2 & k \\ 2 & 1 & 1 \\ 3 & -1 & -k \end{vmatrix}$$

$= 1(-k + 1) + 2(-2k - 3) + k(-2 - 3)$

$= -k + 1 - 4k - 6 - 5k = -10k - 5 = -5(2k + 1)$

$$\Delta_1 = \begin{vmatrix} 1 & -2 & k \\ 2 & 1 & 1 \\ 3 & -1 & -k \end{vmatrix} = -5(2k + 1)$$

$$\Delta_2 = \begin{vmatrix} 1 & 1 & k \\ 2 & 2 & 1 \\ 3 & 3 & -k \end{vmatrix} = 0, \Delta_3 = \begin{vmatrix} 1 & -2 & 1 \\ 2 & 1 & 2 \\ 3 & -1 & 3 \end{vmatrix} = 0$$

$\because z \neq 0 \Rightarrow \Delta = 0$

$\Rightarrow -5(2k + 1) = 0 \Rightarrow k = -\frac{1}{2}$

$\therefore$ System of equation has infinite many solutions.

Let $z = \lambda \neq 0$ then $x = \frac{10 - 3\lambda}{10}$ and $y = -\frac{2\lambda}{5}$

$\therefore$ (x, y) must lie on line $4x - 3y - 4 = 0$

97. **(a)** $\because$ The system of linear equations has a unique solution.

$\therefore \Delta \neq 0$

$$\Delta = \begin{vmatrix} 1+\alpha & \beta & 1 \\ \alpha & 1+\beta & 1 \\ \alpha & \beta & 2 \end{vmatrix} \neq 0$$

$$\Rightarrow \begin{vmatrix} 1+\alpha+\beta+1 & \beta & 1 \\ \alpha+1+\beta+1 & \beta+1 & 1 \\ \alpha+\beta+2 & \beta & 2 \end{vmatrix} \neq 0 \quad [C_1 \to C_1 + C_2 + C_3]$$

$$\Rightarrow (\alpha+\beta+2)\begin{vmatrix} 1 & \beta & 1 \\ 1 & \beta+1 & 1 \\ 1 & \beta & 2 \end{vmatrix} \neq 0$$

$$\Rightarrow (\alpha+\beta+2)\begin{vmatrix} 1 & \beta & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \neq 0 \quad \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$

$\Rightarrow (\alpha + \beta + 2)\,1(1) \neq 0$

$\Rightarrow \alpha + \beta + 2 \neq 0$

$\because$ Ordered pair (2, 4) satisfies this condition

$\therefore \alpha = 2$ and $\beta = 4$.

98. **(a)** Consider the given system of linear equations

$x(1 - \lambda) - 2y - 2z = 0$

$x + (2 - \lambda)y + z = 0$

$-x - y - \lambda z = 0$

Now, for a non-trivial solution, the determinant of coefficient matrix is zero.

$$\begin{vmatrix} 1-\lambda & -2 & -2 \\ 1 & 2-\lambda & 1 \\ -1 & -1 & -\lambda \end{vmatrix} = 0$$

$\Rightarrow \quad (1-\lambda)^3 = 0$

$\lambda = 1$

99. **(b)** $\because$ System of equations has more than one solution

$\therefore \Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$ for infinite solution

$$\Delta_1 = \begin{vmatrix} a & 2 & 3 \\ b & -1 & 5 \\ c & -3 & 2 \end{vmatrix} = a(13) + 2(5c - 2b) + 3(-3b + c)$$

$= 13a - 13b + 13c = 0$

i.e, $\quad a - b + c = 0$

or $\quad b - c - a = 0$

100. **(b)** Since, the system of linear equations has, non-trivial solution then determinant of coefficient matrix = 0

i.e., $\begin{vmatrix} \sin 3\theta & \cos 2\theta & 2 \\ 1 & 3 & 7 \\ -1 & 4 & 7 \end{vmatrix} = 0$

$\sin 3\theta(21 - 28) - \cos 2\theta(7 + 7) + 2(4 + 3) = 0$

$\sin 3\theta + 2\cos 2\theta - 2 = 0$

$3\sin\theta - 4\sin^3\theta + 2 - 4\sin^2\theta - 2 = 0$

$4\sin^3\theta + 4\sin^2\theta - 3\sin\theta = 0$

$\sin\theta(4\sin^2\theta + 4\sin\theta - 3) = 0$

$\sin\theta(4\sin^2\theta + 6\sin\theta - 2\sin\theta - 3) = 0$

$\sin\theta[2\sin\theta(2\sin\theta - 1) + 3(2\sin\theta - 1)] = 0$

$\sin\theta(2\sin\theta - 1)(2\sin\theta + 3) = 0$

$\sin\theta = 0, \sin\theta = \frac{1}{2} \quad \left(\because \sin\theta \neq -\frac{3}{2}\right)$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

Hence, for two values of θ, system of equations has non-trivial solution

101. **(b)** $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & \alpha \end{vmatrix} = 2\alpha - 9 - \alpha + 3 + 1 = \alpha - 5$

$$\Delta_1 = \begin{vmatrix} 5 & 1 & 1 \\ 9 & 2 & 3 \\ \beta & 3 & \alpha \end{vmatrix} = 5(2\alpha - 9) - 1(9\alpha - 3\beta) + (27 - 2\beta)$$

$= 10\alpha - 45 - 9\alpha + 3\beta + 27 - 2\beta$

$= \alpha + \beta - 18$

$$\Delta_2 = \begin{vmatrix} 1 & 5 & 1 \\ 1 & 9 & 3 \\ 1 & \beta & \alpha \end{vmatrix} = 9\alpha - 3\beta - 5(\alpha - 3) + 1(\beta - 9)$$

$= 9\alpha - 3\beta - 5\alpha + 15 + \beta - 9 = 4\alpha - 2\beta + 6$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 5 \\ 1 & 2 & 9 \\ 1 & 3 & \beta \end{vmatrix} = 2\beta - 27 - \beta + 9 + 5 = \beta - 13$$

Since, the system of equations has infinite many solutions. Hence,

$\Delta_1 = \Delta_2 = \Delta_3 = \Delta = 0$

$\Rightarrow \quad \alpha = 5, \beta = 13 \Rightarrow \beta - \alpha = 8$

102. **(c)** Consider the system of linear equations

$x - 4y + 7z = g$ ...(i)

$3y - 5z = h$ ...(ii)

$-2x + 5y - 9z = k$ ...(iii)

Multiply equation (i) by 2 and add equation (i), equation (ii) and equation (iii)

$\Rightarrow \quad 0 = 2g + h + k. \therefore 2g + h + k = 0$

then system of equation is consistent.

103. **(a)** For non zero solution of the system of linear equations;

$$\begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 4 & -3 \end{vmatrix} = 0$$

$\Rightarrow k = 11$

Now equations become

$x + 11y + 3z = 0$ ...(1)

$3x + 11y - 2z = 0$ ...(2)

$2x + 4y - 3z = 0$ ...(3)

Adding equations (1) & (3) we get

$3x + 15y = 0$

$\Rightarrow x = -5y$

Now put $x = -5y$ in equation (1), we get

$-5y + 11y + 3z = 0$

$\Rightarrow z = -2y$

$\therefore \frac{xz}{y^2} = \frac{(-5y)(-2y)}{y^2} = 10$

104. **(c)** Here, the equations are;

$(k + 2)x + 10y = k$

& $kx + (k + 3)y = k - 1$.

These equations can be written in the form of $Ax = B$ as

$$\begin{bmatrix} k+2 & 10 \\ k & k+3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} k \\ k-1 \end{bmatrix}$$

For the system to have no solution

$|A| = 0$

$\Rightarrow \begin{bmatrix} k+2 & 10 \\ k & k+3 \end{bmatrix} = 0 \Rightarrow (k+2)(k+3) - k \times 10 = 0$

$\Rightarrow \ k^2 - 5k + 6 = (k-2)(k-3) = 0$

$\therefore \ k = 2, 3$

For $k = 2$, equations become:

$4x + 10y = 2$

& $2x + 5y = 1$

& hence infinite number of solutions.

For $k = 3$, equations becomes;

$5x + 10y = 3$

$3x + 6y = 2$

& hence no solution.

$\therefore$ required number of values of k is 1

105. (b) The system of linear equations is:

$x + y + z = 2$

$2x + y - z = 3$

$3x + 2y + kz = 4$

As, system has unique solution.

So, $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{vmatrix} \neq 0$

$\Rightarrow k + 2 - (2k+3) + 1 \neq 0$

$\Rightarrow k \neq 0$

Hence, $k \in R - \{0\} \equiv S$

106. (d) As the system of equations has no solution then Δ should be zero and at least one of Δ_1, Δ_2 and Δ_3 should not be zero.

$\therefore \quad \Delta = \begin{vmatrix} 1 & a & 1 \\ 1 & 2 & 2 \\ 1 & 5 & 3 \end{vmatrix} = 0$

$\Rightarrow \ -a - 1 = 0 \Rightarrow a = -1$

$\Delta_2 = \begin{vmatrix} 1 & 3 & 1 \\ 1 & 6 & 2 \\ 1 & b & 3 \end{vmatrix} \neq 0$

$\Rightarrow \ b \neq 9$

107. (a) $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & 1 \\ a & b & 1 \end{vmatrix} = 0$

$\Rightarrow 1[a - b] - 1[1-a] + 1[b - a^2] = 0$

$\Rightarrow (a-1)^2 = 0$

$\Rightarrow a = 1$

For $a = 1$, First two equations are identical

i.e., $x + y + z = 1$

To have no solution with $x + by + z = 0$

$b = 1$

So $b = \{1\} \Rightarrow$ It is singleton set.

108. (b) Since the given system of linear equations has infinitely many solutions.

$\therefore \begin{vmatrix} 2 & 4 & -\lambda \\ 4 & \lambda & 2 \\ \lambda & 2 & 2 \end{vmatrix} = 0$

$\Rightarrow \ \lambda^3 + 4\lambda - 40 = 0$

λ has only 1 real root.

109. (b) For non-trivial solution,

$\begin{vmatrix} 1 & \lambda & -1 \\ \lambda & -1 & -1 \\ 1 & 1 & -\lambda \end{vmatrix} = 0$

$\Rightarrow -\lambda(\lambda+1)(\lambda-1) = 0 \ \Rightarrow \ \lambda = 0, +1, -1$

110. (a) $\left.\begin{array}{r} 2x_1 - 2x_2 + x_3 = \lambda x_1 \\ 2x_1 - 3x_2 + 2x_3 = \lambda x_2 \\ -x_1 + 2x_2 = \lambda x_3 \end{array}\right\}$

$\Rightarrow \ (2-\lambda)x_1 - 2x_2 + x_3 = 0$

$2x_1 - (3+\lambda)x_2 + 2x_3 = 0$

$-x_1 + 2x_2 - \lambda x_3 = 0$

For non-trivial solution,

$\Delta = 0$

i.e. $\begin{vmatrix} 2-\lambda & -2 & 1 \\ 2 & -(3+\lambda) & 2 \\ -1 & 2 & -\lambda \end{vmatrix} = 0$

$\Rightarrow (2-\lambda)[\lambda(3+\lambda) - 4] + 2[-2\lambda + 2] + 1[4 - (3+\lambda)] = 0$

$\Rightarrow \lambda^3 + \lambda^2 - 5\lambda + 3 = 0$

$\Rightarrow \lambda = 1, 1, 3$

Hence λ has 2 values.

111. (b) Given system of equations can be written as

$(a-1)x - y - z = 0$

$-x + (b-1)y - z = 0$

$-x - y + (c-1)z = 0$

For non-trivial solution, we have

$\begin{vmatrix} a-1 & -1 & -1 \\ -1 & b-1 & -1 \\ -1 & -1 & c-1 \end{vmatrix} = 0$

$R_2 \to R_2 - R_3$

$\begin{vmatrix} a-1 & -1 & -1 \\ 0 & b & -c \\ -1 & -1 & c-1 \end{vmatrix} = 0$

$C_2 \to C_2 - C_3$

$\begin{vmatrix} a-1 & 0 & -1 \\ 0 & b+c & -c \\ -1 & -c & c-1 \end{vmatrix} = 0$

Apply $R_3 \to R_3 - R_1$

$$\begin{vmatrix} a-1 & 0 & -1 \\ 0 & b+c & -c \\ -a & -c & c \end{vmatrix} = 0$$

$\Rightarrow (a-1)[bc+c^2-c^2]-1[a(b+c)] = 0$

$\Rightarrow (a-1)[bc]-ab-ac = 0$

$\Rightarrow abc-bc-ab-ac = 0$

$\Rightarrow ab+bc+ca = abc$

112. (b) Since, system of equations have no solution

$\therefore \frac{k+1}{k} = \frac{8}{k+3} \neq \frac{4k}{3k-1}$ ($\because$ System has no solution)

$\Rightarrow k^2+4k+3 = 8k \Rightarrow k^2-4k+3=0$

$\Rightarrow k = 1, 3$

If $k=1$ then $\frac{8}{1+3} \neq \frac{4.1}{2}$ which is false

and if $k=3$ then $\frac{8}{6} \neq \frac{4.3}{9-1}$ which is true, therefore $k=3$

Hence for only one value of k. System has no solution.

113. (b) Given system of equations is homogeneous which is

$x+ay=0$

$y+az=0$

$z+ax=0$

It can be written in matrix form as

$$A = \begin{pmatrix} 1 & a & 0 \\ 0 & 1 & a \\ a & 0 & 1 \end{pmatrix}$$

Now, $|A| = [1-a(-a^2)] = 1+a^3 \neq 0$

So, system has only trivial solution.

Now, $|A| = 0$ only when $a = -1$

So, system of equations has infinitely many solutions which is not possible because it is given that system has a unique solution.

Hence set of all real values of 'a' is $R-\{-1\}$.

114. (c) $\Delta_1 = \begin{vmatrix} 1 & \sin\alpha & \cos\alpha \\ 1 & \cos\alpha & \sin\alpha \\ 1 & -\sin\alpha & \cos\alpha \end{vmatrix}$

$$= \begin{vmatrix} 0 & \sin\alpha-\cos\alpha & \cos\alpha-\sin\alpha \\ 0 & \cos\alpha+\sin\alpha & \sin\alpha-\cos\alpha \\ 1 & -\sin\alpha & \cos\alpha \end{vmatrix}$$

$= (\sin\alpha-\cos\alpha)^2-(\cos^2\alpha-\sin^2\alpha)$

$= \sin^2\alpha+\cos^2\alpha-2\sin\alpha\,.\cos\alpha-\cos^2\alpha+\sin^2\alpha$

$= 2\sin^2\alpha-2\sin\alpha\,.\cos\alpha$

$= 2\sin\alpha(\sin\alpha-\cos\alpha)$

Now, $\sin\alpha-\cos\alpha = 0$ for only

$\alpha = \frac{\pi}{4}$ in $\left(0, \frac{\pi}{2}\right)$

$\therefore \Delta_1 = 2(\sin\alpha)\times 0 = 0,$

since value of $\sin\alpha$ is finite for $\alpha \in \left(0, \frac{\pi}{2}\right)$

Hence non-trivivial solution for only one value of α in $\left(0, \frac{\pi}{2}\right)$

$$\begin{vmatrix} \cos\alpha & \sin\alpha & \cos\alpha \\ \sin\alpha & \cos\alpha & \sin\alpha \\ \cos\alpha & -\sin\alpha & -\cos\alpha \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 0 & \sin\alpha & \cos\alpha \\ 0 & \cos\alpha & \sin\alpha \\ 2\cos\alpha & -\sin\alpha & -\cos\alpha \end{vmatrix} = 0$$

$\Rightarrow 2\cos\alpha(\sin^2\alpha-\cos^2\alpha) = 0$

$\therefore \cos\alpha = 0$ or $\sin^2\alpha-\cos^2\alpha = 0$

But $\cos\alpha = 0$ not possible for any value of $\alpha \in \left(0, \frac{\pi}{2}\right)$

$\therefore \sin^2\alpha-\cos^2\alpha = 0 \Rightarrow \sin\alpha = -\cos\alpha$, which is also not possible for any value of $\alpha \in \left(0, \frac{\pi}{2}\right)$

Hence, there is no solution.

115. (d) Given system of equations can be written in matrix form as AX = B where

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 2 & 5 & a \end{pmatrix} \text{ and } B = \begin{pmatrix} 6 \\ 9 \\ b \end{pmatrix}$$

Since, system is consistent and has infinitely many solutions

$\therefore$ (adj. A) B = 0

$$\Rightarrow \begin{pmatrix} 3a-25 & 15-2a & 1 \\ 10-a & a-6 & -2 \\ -1 & -1 & 1 \end{pmatrix}\begin{pmatrix} 6 \\ 9 \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$\Rightarrow -6-9+b=0 \Rightarrow b=15$

and $6(10-a)+9(a-6)-2(b)=0$

$\Rightarrow 60-6a+9a-54-30=0$

$\Rightarrow 3a=24 \Rightarrow a=8$

Hence, $a=8, b=15$.

116. (a) Given system of equations is

$x+ky+3z=0$

$3x+ky-2z=0$

$2x+3y-4z=0$

Since, system has non-trivial solution

$$\therefore \begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 3 & -4 \end{vmatrix} = 0$$

$\Rightarrow \quad 1(-4k+6)-k(-12+4)+3(9-2k)=0$

$\Rightarrow \quad 4k+33-6k=0 \Rightarrow k=\frac{33}{2}$

Hence, statement - 1 is false.
Statement-2 is the property.
It is a true statement.

117. (d) Given system of equations is

$x+y+z=6$
$x+2y+3z=10$
$x+2y+\lambda z=0$

It has unique solution.

$\therefore \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{vmatrix} \neq 0$

$\Rightarrow \quad 1(2\lambda-6)-1(\lambda-3)+1(2-2)\neq 0$

$\Rightarrow \quad 2\lambda-6-\lambda+3\neq 0 \Rightarrow \lambda-3\neq 0 \Rightarrow \lambda \neq 3$

118. (a) $x-ky+z=0$

$kx+3y-kz=0$
$3x+y-z=0$

The given that system of equations have trivial solution,

$\therefore \begin{vmatrix} 1 & -k & 1 \\ k & 3 & -k \\ 3 & 1 & -1 \end{vmatrix} \neq 0$

$\Rightarrow \quad 1(-3+k)+k(-k+3k)+1(k-9)\neq 0$

$\Rightarrow \quad k-3+2k^2+k-9\neq 0$

$\Rightarrow \quad k^2+k-6\neq 0 \quad \Rightarrow \quad k=-3, k\neq 2$

So, the equation will have only trivial solution, when $k \in R-\{2,-3\}$

119. (a) Given that system of equations have non-zero solution $\Delta=0$

$\Rightarrow \begin{vmatrix} 4 & k & 2 \\ k & 4 & 1 \\ 2 & 2 & 1 \end{vmatrix} = 0$

$\Rightarrow \quad 4(4-2)-k(k-2)+2(2k-8)=0$

$\Rightarrow \quad 8-k^2+2k+4k-16=0$

$k^2-6k+8=0$

$\Rightarrow (k-4)(k-2)=0 \Rightarrow k=4,2$

120. (c) $D=\begin{vmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & 5 & 2 \end{vmatrix}=0$

$D_x=\begin{vmatrix} 3 & 2 & 1 \\ 3 & 3 & 1 \\ 1 & 5 & 2 \end{vmatrix}\neq 0$

$\Rightarrow$ Given system, does not have any solution.
$\Rightarrow$ No solution

121. (d) The given equations are

$-x+cy+bz=0$
$cx-y+az=0$
$bx+ay-z=0$

Given that x, y, z are not all zero

$\therefore$ The above system have non-zero solution

$\Rightarrow \Delta=0 \Rightarrow \begin{vmatrix} -1 & c & b \\ c & -1 & a \\ b & a & -1 \end{vmatrix}=0$

$\Rightarrow -1(1-a^2)-c(-c-ab)+b(ac+b)=0$
$\Rightarrow -1+a^2+b^2+c^2+2abc=0$
$\Rightarrow a^2+b^2+c^2+2abc=1$

122. (a) $\alpha x+y+z=\alpha-1;$
$x+\alpha y+z=\alpha-1;$
$x+y+z\alpha=\alpha-1$

$\Delta=\begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{vmatrix}$

$=\alpha(\alpha^2-1)-1(\alpha-1)+1(1-\alpha)$
$=\alpha(\alpha-1)(\alpha+1)-1(\alpha-1)-1(\alpha-1)$
$=(\alpha-1)[\alpha^2+\alpha-1-1]$
$=(\alpha-1)[\alpha^2+\alpha-2]$
$=(\alpha-1)[\alpha^2+2\alpha-\alpha-2]$
$=(\alpha-1)[\alpha(\alpha+2)-1(\alpha+2)]$
$=(\alpha-1)^2(\alpha+2)$

$\because$ Equations has infinite solutions
$\therefore$ $\Delta=0$
$\Rightarrow$ $(\alpha-1)=0,\ \alpha+2=0$
$\Rightarrow$ $\alpha=-2, 1;$

But $\alpha\neq 1$.
$\therefore \alpha=-2$

123. (d) For homogeneous system of equations to have non zero solution, $\Delta=0$

$\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix}=0$

Applying $C_2 \to C_2-2C_3$

$\Rightarrow \begin{vmatrix} 1 & 0 & a \\ 1 & b & b \\ 1 & 2c & c \end{vmatrix}=0 \; R_3\to R_3-R_1, R_2\to R_2-R_1$

$\Rightarrow \begin{vmatrix} 1 & 0 & a \\ 0 & b & b-a \\ 0 & 2c & c-a \end{vmatrix}=0$

$\Rightarrow bc-ab=2bc-2ac$

$\Rightarrow \frac{2}{b}=\frac{1}{a}+\frac{1}{c}$

$\therefore a,b,c$ are in Harmonic Progression.